Population Structure and Asset Values

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Abstract

With the large baby-boom cohort entering retirement, many are concerned that the expected drop in saving and investment will result in substantially diminished asset prices and compromised pension plans. This paper contributes to the quantification of the link between population structure and asset values, by generating returns on assets in the presence of demographic change. We carry this out in the context of a large scale computable Overlapping Generations Model (OLG) with endogenous labor supply, aggregate risk, and two asset classes. Our model generates typical age specific asset holdings and consumption patterns, and results in age specific portfolio allocations consistent with the data. We use counterfactuals to predict the outcome of changes in demographic structure, and find that asset prices are moderately lower with an older population. Specifically, a 4% increase in the survival probability of households over age 65 results in a 4.16% drop in the return on capital, and a 3.02% drop in the return on bonds. While our baseline model employs a two-pillar pension system (a pay-go public provision plus private saving), we also explore a three-pillar pension system (adding a publicly administered, partially funded, employment-related plan) and consider the implications of tax and pension policy on economic outcomes. This framework will be helpful to assess the implications of the proposed expansion of the Canada Pension Plan. Additional modifications include a bequest motive and age dependent health care costs.

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1 Introduction

The large baby-boom cohort, which has affected economic growth for six decades, has just started to enter retirement. With a higher old age dependency ratio, economies may expect significant implications for the asset market, the labour market and long term growth. Specifically, a major concern is that with an aging population, there will be less saving and investment (and a shift in asset allocation), which could severely diminish asset prices. These concerns have spurred research on the impact of population aging on asset prices (e.g. Mankiw and Weil, 1989; Börsch-Supan et al., 2006; Cornell, 2012; Kang, 2013). In addition to concerns over asset prices, large-scale retirement is associated with reduced labour force growth, and with dissaving (Beach, 2008). Policy makers and academics alike have raised concerns over the ability of pension plans and private savings in Canada to meet comfortable retirement income targets (Ambachtsheer, 2009; Horner, 2009). The ability of savings to meet targets could be further compromised in the event of depressed asset prices, or an asset price meltdown.[1]

This paper, seeks to quantify the impact of population structure on asset values and to consider the effects of different tax and pension parameters on pension outcomes. Because the high old age-dependency ratio is at the heart of policy makers concerns, we develop a computable Overlapping Generations Model (OLG) to explore the implications of an older demographic structure on economic outcomes. We calibrate and simulate a 20-period OLG life-cycle model with endogenous labour supply, aggregate uncertainty, two asset classes (risky and risk-free), and a simple two pillar pension system (a public pay-go plan, and private savings). This model generates typical age specific asset holdings and consumption patterns, and results in age specific portfolio allocations consistent with the data. We construct counterfactuals to consider what happens if the population structure is altered, and find that asset prices are moderately lower with an older population. Specifically, a 4% increase in the survival probability of households over age 65 results in a 4.16% drop in the return on capital, and a 3.02% drop in the return on bonds.

Within the retirement literature, there is also an ongoing discussion about what policies could secure better retirement prospects for Canadians (Ambachtsheer, 2009). Ideas

[1]However, we note that there is little consensus in the literature on the potential impact of population aging on asset values. Study conclusions range from catastrophic impacts, described as "asset meltdown" (Brook, 1998; Mankiw and Weil, 1989; Kang, 2013); to moderate effect on the markets but no meltdown (Abel, 2003; Andrews et al., 2014; Börsch-Supan et al., 2006; Börsch-Supan, 2006; Campbell, 2001; Geanakoplos et al., 2004; Liu and Spiegel, 2011; Poterba, 2004; Schieber and Shoven, 1994); to a total rejection of the asset meltdown hypothesis (Kedar-Levy, 2006; Green and Hendershott, 1996; Cornell, 2012; Bovbjerg and Scott, 2006).

range from lower tax rates on investments to stimulate saving (for example, tax-preferred accounts with low or deferred tax rates), combinations of defined benefit and defined contribution structures, increased contributions to the publicly administered employment related Canada/Quebec Pension Plans,[2] and increased outlays of the pay-as-you-go public pension system (Old Age Security and Guaranteed Income Supplement). These policy tools could influence the saving and investment decisions of households as well as market returns and economic growth.

As such, we explore alternative pension systems within the framework of our model. Our baseline model employes a two-pillar pension system (a pay-go public provision plus private saving), subsequently we explore a three-pillar pension system by adding a publicly administered, partially funded, employment-related plan. Compared to the two-pillar system, the three-pillar system generates lower private investment, and reduced total asset holding among older cohorts, but has a smaller impact on consumption. Our counterfactual exercises result in similar outcomes to the two pillar model. That is, an older population generates moderately lower asset prices. Interestingly, when we target a higher income replacement ratio for the employment-related pension plan, there is an apparent decrease in households' total asset holding, which results in an increase in asset returns. The capital holding moves towards younger generations.

This paper makes three specific contributions to the literature. First, while a handful of empirical studies have found associations between demographics and specific asset classes (Ang and Maddaloni, 2003; Bakshi and Chen, 1994; Goyal, 2004; Poterba, 2001; Poterba, 2004), few studies integrate demographic structure in a model with more than one asset class (see Brooks, 2000; Bucciol and Beetsma, 2011; Černỳ et al., 2006; Hasanhodzic and Kotlikoff, 2015; Muto et al., 2012; Reiter, 2015; Xu, 2013). Incorporating both a risk-free and a risky asset, with endogenous returns, will help us better estimate the effect of a high old age dependency ratio on asset prices, and on the financial health of various cohorts within the population. To the best of our knowledge, this has not been done previously.

Second, our baseline model employs a standard two-pillar pension system; however we then expand this framework to test the implications of demographic structure under a three-pillar pension system which includes a publicly administered, partially funded, employment-related pension. This framework will be helpful to assess the implications of the proposed expansion of the Canada Pension Plan. Finally, we devote considerable

[2] Indeed, in 2016, the Canada Revenue Agency announced changes to the Canada Pension Plan that aim to increase the targetted replacement rate from 25% to 33% of pre-retirement earnings. This increase will be funded by higher contribution rates as well as an increase in the maximum pensionable earnings.

attention to generating age-specific portfolio allocations that are consistent with the data. Our model does a reasonable job in this regard, with the exception of the oldest cohort, and we explore additional mechanisms to address this concern (e.g. bequest motive, health care costs and, as a next step, intra-cohort heterogeneity).

2 Computational Methods and Advancing Complexity in the OLG Framework

The Overlapping Generations (OLG) model framework is well suited, and frequently used, to analyze the effects of demographic structure because it incorporates multiple generations. Each generation has different incentives at each stage of life, and changes in the relative size of these generations will affect the amount of consumption and saving at any period of time. Saving and consumption decisions of each cohort will also be sensitive to the policy environment, making the model attractive for policy analysis. However, simplifying assumptions and different calibrations can dramatically change the quantitative predictions of these models. Simplifications, while necessary for a tractable model, can miss important implications of the demographic shift. For example, few studies have integrateed demographic structure in a model with more than one asset class, yet incorporating both a risk-free and a risky asset, with endogenous returns, will help us better estimate the effect of a high old age dependency ratio on asset prices.

Advanced computational methods make it possible to integrate some of these complexities without losing tractability. The quantitative analysis of deterministic OLG models with a large number of cohorts has been possible since Auerbach and Kotlikoff (1983) and Auerbach and Kotlikoff (1987). But OLG models with both aggregate uncertainty and long-lived agents still pose difficult computational challenges due to the curse of dimensionality because the number of state variables dramatically increases with the number of generations.

A possible candidate for the approximation of equilibria in stochastic OLG economies with a large number of generations is using Krusell and Smith's low-dimensional approximation approach. Krusell and Smith (1998) introduce a framework that is designed for modelling the aggregate behaviour of a very large number of agents that are identical in fundamentals. They replace the true state space with a reduced space, such as the first several moments of the true space. Gourinchas et al. (2000) and Storesletten et al. (2007) use the same approach to approximate equilibria in stochastic OLG models. However, Krueger and Kubler (2004) point out that using Krusell and Smith's approach in OLG models often leads to very high relative errors in agents' Euler equations. This is because in OLG models, agents are typically different in fundamentals, in the sense that households' propensity to save varies greatly by age. Instead, Krueger and Kubler (2004) use a technique, based on the Smolyak (1963) algorithm, that efficiently chooses grid values to approximate over a sparse set of points in the entire state space. Of course, this method cannot fully overcome the curse of dimensionality. Indeed, Krueger and Kubler (2004) limit their model to 10 generations for computational feasibility.

This paper follows the methodology used in Hasanhodzic and Kotlikoff (2015), which use a simulation approach in the spirit of the generalized stochastic simulation algorithm (GSSA) by Judd et al. (2009, 2011). GSSA is built on the Marcet (1988) insight that economic behaviour needs to be calculated only for states that the economy will actually reach with nontrivial probability. Hasanhodzic and Kotlikoff (2015) simulate an economy with capital and bonds and 80 cohorts. They use the vector of household net worth as state variables and approximate household consumption functions by globally linear functions. They find that generational risk is small but that Social Security can exacerbate it. However, Hasanhodzic and Kotlikoff (2015) do not characterize demographic structure, e.g. there is no population growth, no accidental death, and no intra-cohort heterogeneity. Therefore, their paper is silent in studying the impacts of demographic changes on the economy. Reiter (2015) provides another computationally efficient method to solve overlapping generations models with asset choice. He shows that a global projection method with polynomial approximations of degree 3 is sufficient to provide a very precise solution, even in the case of large shocks. However, global linear approximations are not sufficient to pin down households' asset choices reliably. Similar to Hasanhodzic and Kotlikoff (2015), Reiter (2015) does not take population variables into consideration.

In this paper, we calibrate and simulate a 20-period overlapping generations (OLG) life-cycle models with aggregate productivity shocks. Our model can be further characterized by the following features. First, households can save and invest in two financial assets: one is a one-period risk-free bond and the other is risky capital (stock). Second, we incorporate population growth, mortility and accidental bequest to study the links between demographic structure and economic activities. Third, with endogenous labour supply, we can examine how households provide labour in response to economic shocks. Fourth, for the purpose of studying real economies, we introduce taxes on consumption, labour, investment and pension incomes. Finally, the effects of fertility and education on individual behaviour are also considered, in terms of reductions in households' time endowment. We use Canadian data to parameterize the model.

The base model includes a pay-go pension system. A second section will additionally include a partially funded pension. The pension payment and government's portfolio of this system are both exogenous. We calibrate (on public asset allocation) the model such that at the steady state, the magnitude of the pay-go versus funded system approximates the country's current levels of pay-go versus funded relative to the size of the economy. We then contrast results for this expanded pension model under different replacement rates (to investigate impact of CPP expansion).

3 Model environment

3.1 Demographics

Time is discrete and goes forever. During each time period, the household sector is made of J overlapping generations, of age between 18 and 97. We use $j \in \{1, 2, \ldots, J\}$ to denote cohorts' age. Moreover, households are generally categoried into five life stages: young-working (YW), middle-working (MW), mature-working (W), semi-retired (SR), and retirement (R), corresponding to age groups $\{18-33, 34-49, 50-65, 66-81, 82-97\}$ respectively.

Let $N_{j,t}$ represent the size of generation j in period t. In our baseline model, there is no heterogeneity within each cohort, later versions of our model will incorporate heterogeneity (e.g. in terms of gender, productivity and education). We use a representive household, which has a size of $N^i_{j,t}$, to characterize type i households at age j in period t, where $i \in \{1, \ldots, I\}$. In the model with no intra-cohort heterogeneity, $i = 1$ and is representative of the average household of age j in period t.

In each period t, a new generation aged $j = 1$ is born into the economy, while the other existing generations each shifts forward by one. The exogenous growth rate of the new generation $j = 1$ is denoted by n. Each type i household at age j has an exogenous marginal probability ϕ^i_j of reaching age $j+1$ in period $t+1$. The oldest generation, $j = J$, dies out deterministically in the subsequent period, i.e. $\phi^i_J = 0$. Then, the demographic stucture in period t is expressed as below:

$$N^i_{j,t} = \begin{cases} (1+n)\chi^i N_{0,t-1}, & \text{if } j = 1, \\ \phi^i_{j-1}\chi^i N_{j-1,t-1}, & \text{if } 1 < j \leq J. \end{cases}$$

where χ^i is the proportion of type i households within a generation. In our basic model,

χ^i is constant across generations.

3.2 Households

At each age, each household has a fixed constant H units of time to spend on labor and leisure. In addition, at their YW and MW stages, a household at age j mandatorily spends FC_j percent of H units of time per period on fertility (which can be thought of as time required for child rearing), respectively. Similarly, the household is required to take FE_j percent of H units of time on education. Let H_j denote the total available time that can be allocated between labour and leisure for households at age j.

$$H_j = \begin{cases} H\left(1 - FC_j - FE_j\right), & \text{if } j \in \{YW, MW\}, \\ H, & \text{if } j \in \{W, SR, R\}. \end{cases} \tag{1}$$

In all working ages, a household decides how much labour to supply to firms and earns wage income according to its labor efficiency ε_j^i, which is exogenously given. Starting from the SR stage, the household receives pension income. At its SR stage, in addition to receiving the pension, the household determines how much labour to supply out of a restricted $\iota_p H$ units of time. Thus, ι_p is the maximum fraction of the period that an SR household may work. Retirees supply zero labour and enjoy all available time as leisure with pension income.

Households value both consumption and leisure according to the following periodic utility function:

$$u^i\left(c, h\right) = \frac{c^{1-\gamma_c^i}}{1-\gamma_c^i} + \Psi^i \frac{\left(H_j - h\right)^{1-\gamma_h^i}}{1-\gamma_h^i}$$

where c and h denote consumption and labour supplied, respectively. γ_c^i represents the relative risk-aversion and γ_h^i represents the parameter that regulates the Frisch elasticity of labour supply. Ψ^i represents the utility weight of leisure relative to market consumption.

Following Neusser (1993), we assume that for the oldest cohort, leaving wealth to other generations generates utility irrespective of the well-being of the heirs. This is the simplest way of introducing a bequest motive in a model in which no such motive can arise endogenously. We introduce this motive to more closedly match observed asset holding at end of life.

3.3 Assets

Households can save and invest in two financial assets: one is a one-period risk-free bond and the other is risky capital (stock). Let $\theta_{j,t}^i$ denote a household's total demand for assets (savings) and $\eta_{j,t}^i$ the share of saving invested in risk-free bonds at the end of period t. There is neither a borrowing constraint on bonds nor a short sale constraint on stock in the basic model. Households who invest one unit of consumption in bonds in period t receive $1 + \bar{r}_t$ units in period $t + 1$with certainty. Note $\bar{r}_t$ is known in period t although it is received in the next period. On the other hand, the return of one unit of consumption invested in capital in period t is r_{t+1}, which is realized in period $t + 1$. Households enter period t with $\theta_{j-1,t-1}$ in assets, which corresponds to the total assets they demanded in the prior period.

Holding risk-free assets can be negative, which reflects the fact that households may borrow. In this basic model, risk-free bonds are in zero net supply, therefore we have:

$$\sum_j \sum_i \eta_{j,t}^i \theta_{j,t}^i N_{j,t}^i = 0. \tag{2}$$

Because households' investment decisions are made at the end of each period, the total capital used in production in period t, K_t, is given by

$$K_t = \sum_j \sum_i \left(1 - \eta_{j,t-1}^i\right) \theta_{j,t-1}^i N_{j,t-1}^i. \tag{3}$$

New born young workers $j = 1$ enter the economy with zero asset holding, i.e. $\theta_{0,t}^i = 0$. The oldest generation consumes and leaves the economy with asset holding as voluntary bequest.

3.4 Production

In each period, a representative firm uses labour H_t, in efficient units, and physical capital K_t to produce total final goods Y_t. We assume a Cobb-Douglas production function and no adjustment cost on capital:

$$Y_t = z_t K_t^\alpha H_t^{1-\alpha},$$

where $\alpha \in (0, 1)$ is the capital share.

The total factor productivity (TFP) z_t follows a simple AR(1) process:

$$\ln(z_t) = \rho \ln(z_{t-1}) + \nu_t,$$

where $\nu_t \sim N(0, \sigma_z^2)$.

The investment-specific technology shock follows a simple AR(1) process.

$$K_t = (1-\delta)K_t + q_t I_t,$$

$$In(q_t) = \rho_t In(q_{t-1}) + v_{q,t},$$

where $v_{q,t} \sim N(0, \sigma_t^2), \sigma_z$ is uncorrelated with σ_q.

The aggregate amount of efficient labor in period t, H_t, is given by:

$$H_t = \sum_j \sum_i \varepsilon_j^i h_{j,t}^i N_{j,t}^i, \tag{4}$$

where ε_j^i represents age- and type-specific labour productivity. Therefore, $\varepsilon_j^i h_{j,t}^i$ is the efficient labour supplied by a type i household at age j in period t.

3.5 Government

3.5.1 2-pillar pension system

We first consider a two-pillar pension system. In addition to households' private saving, old households get funds from a pay-as-you-go proportional pension scheme. For the pay-as-you-go scheme, the government takes a fixed percentage, τ_s, of wage from each current worker, and this income is distributed uniformly among the retirees. Let p_t represent the pension income for a retiree in period t:

$$p_{j,t} = \begin{cases} 0, & \text{if } j \in \{YW, MW, W\}, \\ \frac{\tau_s w_t H_t}{\sum_{j \in \{OW,R\}} \sum_i N_{j,t}^i}, & \text{if } j \in \{SR, R\}, \end{cases} \tag{5}$$

where w_t is the wage rate paid by the firm in period t.

3.5.2 3-pillar pension system

Now on top of the above 2-pillar pension system, we introduce a partly funded pension system. For this system, we assume the government holds a pool of assets θ_G, with η_G proportion of risk-free bonds and $(1-\eta_G)$ proportion of risky capital. Note here (θ_G, η_G)

are both exogenously given, i.e. we choose them to match related targets (set θ_G to be 95% of steay state GDP, and η_G to be 40%). Every period, the government pays the income from its asset holdings to the households, plus, imposes whatever tax needed on the working cohorts in order to pay out ratio κ_j of pre-retirement income to the retired cohorts. At this moment, we set the payout to be exactly κ_j percent of the average wage income of the age $j = SR - 1$ generation at the steady state. We think this is reasonable because we use stationary population structure here and the economy just fluctuates around the steady state. Note κ_j is a flat rate (50%) for all retirees. Therefore we have the funded pension payout as

$$p_j^G = \begin{cases} 0, & \text{if } j \in \{YW, MW, W\}, \\ \kappa_j \left(\frac{w_{SS} \sum_i \varepsilon_{SR-1}^i h_{SR-1,SS}^i N_{SR-1,SS}^i}{\sum_i N_{SR-1,SS}^i} \right), & \text{if } j \in \{SR, R\}, \end{cases} \tag{6}$$

and the government's budget (for the funded pension system) as

$$\sum_{j=SR}^{W} p_j^G N_{j,t}^i = \left[\eta_G \left(1 + (1 - \tau_r) \bar{r}_{t-1}\right) + (1 - \eta_G) \left(1 + (1 - \tau_r) r_t\right)\right] \theta_G + \tau_{s,t}^G w_t H_t, \tag{7}$$

where $\tau_{s,t}^G$ is therefore the endogenous tax rate imposed on working cohorts in order to pay out funded pension. Note if the income from θ_G is large enough to payout the required pension income,$\tau_{s,t}^G$ is negative, i.e. a tax deduction on labour income. Moreover, if there is population growth, the government needs to maintain θ_G such that it grows at the same pace with the total population.

With the partly funded pension, we modify aggregate assets holdings (2) and (3) as follows.

$$\sum_j \sum_i \eta_{j,t}^i \theta_{j,t}^i N_{j,t}^i + \eta_G \theta_G = 0,$$

$$K_t = \sum_j \sum_i \left(1 - \eta_{j,t-1}^i\right) \theta_{j,t-1}^i N_{j,t-1}^i + (1 - \eta_g) \theta_G.$$

3.5.3 Other taxes and accidental bequest

The government also collects taxes from households to be spent on other items, which are not modelled here. These taxes include a proportional consumption tax (τ_c), an investment tax (τ_r), and a tax on pension income (τ_p).

If a household dies accidentally before the highest age J, its net wealth is collected by the government rather than being inherited. The government collects all residual assets from the fraction of the population that dies and transfers this sum equally to all remaining households. Let ξ_t be the lump-sum transfer associated with accidental bequests that are left by households who die at the end of period $t-1$.

$$\xi_t = \frac{\sum_j \sum_i \left(1-\phi^i_{j-1}\right)\left[\eta^i_{j-1,t-1}\left(1+\left(1-\tau_r\right)\bar{r}_{t-1}\right)+\left(1-\eta^i_{j-1.t-1}\right)\left(1+\left(1-\tau_r\right)r_t\right)\right]\theta^i_{j-1,t-1}N^i_{j-1,t-1}}{\sum_j \sum_i N^i_{j,t}}$$

4 Agent problems

4.1 Household decisions

The timing of household decisions is as follows. At the beginning of each period t, a type i age j representative household holds assets $\theta^i_{j-1,t-1}$, which are brought from period $t-1$. During the period, the household supplies labor, $h^i_{j,t}$, to the firm and earns an income commensurate with their efficient hours and the market wage. At the end of period t, the household's total available resources include the gross return on risk-free bonds and risky capital, wage income, and pension income, less taxes. Then the household decides how to allocate these resources on consumption, $c^i_{j,t}$, asset holdings for the next period, $\theta^i_{j,t}$, and the share of investment on bonds, $\eta^i_{j,t}$. Deaths occur at the end of the period and the residual assets from the fraction of the population that died are collected by the government.

The state of the economy is given by $(s_t; z_t) = \left(x^1_{2,t}, \cdots, x^i_{j,t}, \cdots, x^I_{J,t}; z_t\right)$, where $x^i_{j,t}$ is the value of asset holding of a representative type i households at age j in period t :

$$x^i_{j,t} = \left[\eta^i_{j-1,t-1}\left(1+\left(1-\tau_r\right)\bar{r}_{t-1}\right)+\left(1-\eta^i_{j-1,t-1}\right)\left(1+\left(1-\tau_r\right)r_t\right)\right]\theta^i_{j-1,t-1} \tag{8}$$

Note $x^i_{1,t}=0$ since newly born young workers enter the economy with zero asset holding.

Let $V^i_j(s_t; z_t)$ be the value of the representative household:

$$V^i_j(s_t; z_t) = \max_{\left\{c^i_{j,t}, h^i_{j,t}, \theta^i_{j,t}, \eta^i_{j,t}\right\}} u^i\left(c^i_{j,t}, h^i_{j,t}\right) + \beta\phi^i_j E_t\left[V^i_{j+1}\left(s_{t+1}; z_{t+1}\right)\right] \tag{9}$$

subject to the following budget constraint:

$$(1+\tau_c)\,c^i_{j,t}+\theta^i_{j,t}\leq\left\{\ \left(1-\tau_h-\tau^G_{s,t}\right)w_t\varepsilon^i_j h^i_{j,t}+x^i_{j,t}+(1-\tau_p)\left(p_{j,t}+p^G_j\right)+\xi_t\ \right\}. \tag{10}$$

and the time constraint of labour:

$$h^i_{j,t}\leq H^c_j=\begin{cases}H_j, & \text{if } j\in\{YW,MW,W\}\,,\\ \iota_p H, & \text{if } j\in\{SR\}\,,\\ 0, & \text{if } j\in\{R\}\,,\end{cases} \tag{11}$$

β is the households' discount factor. Households of the oldest generation, $j=J$, have the following problem.

$$V^i_J(s_t;z_t)=\max_{\left\{c^i_{J,t},\theta^i_{J,t},\eta^i_{J,t}\right\}} u^i\left(c^i_{J,t},0\right)+\beta E_t\left[v^i\left(X^i_{J+1,t+1}\right)\right]$$

where

$$\begin{aligned}X^i_{J+1,t+1}&=\left[\eta^i_{J,t}\left(1+(1-\tau_r)\,\bar{r}_t\right)+\left(1-\eta^i_{J,t}\right)\left(1+(1-\tau_r)\,r_{t+1}\right)\right]\theta^i_{J,t}\\ v\left(X\right)&=\Gamma\frac{X^{1-\gamma^i_c}}{1-\gamma^i_c}\end{aligned}$$

Γ denotes the intensity of the bequest motive. To make the analysis simple, it is assumed that wealth is distributed equally (the same as incidental bequest) to all existing cohorts.

For generations $j<J$, the first order conditions (FOCs) with respect to the four control variables, $\left\{c^i_{j,t},h^i_{j,t},\theta^i_{j,t},\eta^i_{j,t}\right\}$ are given as follows:

$$u^i_1\left(c^i_{j,t},h^i_{j,t}\right)-\lambda^1_{j,t}\left(1+\tau_c\right)=0, \tag{12}$$

$$u^i_2\left(c^i_{j,t},h^i_{j,t}\right)+\lambda^1_{j,t}\left(1-\tau_h-\tau^G_{s,t}\right)w_t\varepsilon^i_j-\lambda^2_{j,t}=0, \tag{13}$$

$$\beta\phi^i_j E_t\left[\frac{\partial V^i_{j+1}\left(s_{t+1};z_{t+1}\right)}{\partial\theta^i_{j,t}}\right]-\lambda^1_{j,t}=0, \tag{14}$$

$$\beta\phi^i_j E_t\left[\frac{\partial V^i_{j+1}\left(s_{t+1};z_{t+1}\right)}{\partial\eta^i_{j,t}}\right]=0, \tag{15}$$

where $\lambda^1_{j,t}$ and $\lambda^2_{j,t}$ are the Lagrange multipliers for budget and time constraints, respectively.

As for $j = J$, the FOCs are

$$\begin{aligned}
u_1^i\left(c_{J,t}^i, 0\right) - \lambda_{J,t}^1\left(1+\tau_c\right) &= 0, \\
\beta E_t\left[\Gamma\left(X_{J+1,t+1}^i\right)^{-\gamma_c^i}\left[\eta_{J,t}^i\left(1+\left(1-\tau_r\right)\bar{r}_t\right)+\left(1-\eta_{J,t}^i\right)\left(1+\left(1-\tau_r\right)r_{t+1}\right)\right]\right] &= \lambda_{J,t}^1, \\
\beta E_t\left[\Gamma\left(X_{J+1,t+1}^i\right)^{-\gamma_c^i}\left(1-\tau_r\right)\left(\bar{r}_t - r_{t+1}\right)\theta_{J,t}^i\right] &= 0,
\end{aligned}$$

Envelope Theory implies

$$\frac{\partial V_{j+1}\left(s_{t+1};z_{t+1}\right)}{\partial \theta_{j,t}^i} = \lambda_{j+1,t+1}^1\left[\eta_{j,t}^i\left(1+\left(1-\tau_r\right)\bar{r}_t\right)+\left(1-\eta_{j,t}^i\right)\left(1+\left(1-\tau_r\right)r_{t+1}\right)\right], \quad (16)$$

$$\frac{\partial V_{j+1}^i\left(s_{t+1};z_{t+1}\right)}{\partial \eta_{j,t}^i} = \lambda_{j+1,t+1}^1\left(1-\tau_r\right)\left(\bar{r}_t - r_{t+1}\right)\theta_{j,t}^i. \quad (17)$$

Then substituting (16) and (17) into (14) and (15), and using (12) to eliminate $\lambda_{j,t}^1$, we get the following non-linear equation system that solves the households' problems.

$$\left(c_{j,t}^i\right)^{-\gamma_c} = \beta\phi_j^i E_t\left[\left(1+\left(1-\tau_r\right)r_{t+1}\right)\left(c_{j+1,t+1}^i\right)^{-\gamma_c}\right], \quad (18)$$

$$0 = \beta\phi_j^i E_t\left[\left(1-\tau_r\right)\left(\bar{r}_t - r_{t+1}\right)\left(c_{j+1,t+1}^i\right)^{-\gamma_c}\right], \quad (19)$$

$$\frac{\Psi^i\left(H_j - h_{j,t}^i\right)^{-\gamma_h} + \lambda_{j,t}^2}{\left(c_{j,t}^i\right)^{-\gamma_c}} = \frac{1-\tau_h-\tau_{s,t}^G}{1+\tau_c} w_t \varepsilon_j^i. \quad (20)$$

$$\lambda_{j,t}^2\left(H_j^c - h_{j,t}^i\right) = 0 \quad (21)$$

otherwhere E_t is the conditional expectation of z_{t+1} given z_t. (18) and (19) are Euler equations that characterize returns on risk-free bonds and risky capital. Equation (20) indicates the intra-temporal substitution between consumption and labor supply. Equation (21) is the complementary slackness condition. Note for $j = J$, (18) and (19) are replaced by the following two equations.

$$\begin{aligned}
\beta\Gamma E_t\left[\left(1+\left(1-\tau_r\right)r_{t+1}\right)\left(X_{J+1,t+1}^i\right)^{-\gamma_c^i}\right] &= \left(c_{J,t}^i\right)^{-\gamma_c^i}, \\
\beta\Gamma E_t\left[\left(1-\tau_r\right)\left(\bar{r}_t - r_{t+1}\right)\left(X_{J+1,t+1}^i\right)^{-\gamma_c^i}\right] &= 0.
\end{aligned}$$

4.2 Firm decisions

The profit-maximizing behavior of the firm gives rise to first order conditions that determine the real net-of-depreciation rate of return to capital and the real wage rate per unit of efficient labour, respectively:

$$r_t = \alpha z_t K_t^{\alpha-1} H_t^{1-\alpha} - \delta, \tag{22}$$

$$w_t = (1-\alpha) z_t K_t^{\alpha} H_t^{-\alpha}. \tag{23}$$

where $\delta \in [0,1]$ is the depreciation rate.

5 Recursive competitive equilibrium

At the beginning of each period, the state of the economy is given by $\{s_t; z_t\}$, where $s_t = \left(x_{2,t}^1, \ldots x_{j,t}^i, \ldots x_{J,t}^I\right)$ represents the distribution of values of asset holdings in period t. Given the initial state of the economy $(s_0; z_0) = \left(x_{2,0}^1, \ldots, x_{j,0}^i, \ldots, x_{J,0}^I; z_0\right)$, the recursive competitive equilibrium is defined as follows:

Definition: the **Recursive Competitive Equilibrium (RCE)** consists of value functions $V_j^i(s_t; z_t)$; the household policy functions for consumption $c_{j,t}^i(s_t; z_t)$, labor supply $h_{j,t}^i(s_t; z_t)$, total saving $\theta_{j,t}^i(s_t; z_t)$, and the share of savings invested in risk-free bonds $\eta_{j,t}^i(s_t; z_t)$ for each age and type (j, i); the inputs for the representive firm $K_t(s_t; z_t)$ and $H_t(s_t; z_t)$; the government policy $p_t(s_t; z_t)$, $\tau_{s,t}^G(s_t; z_t)$; and prices $\bar{r}_t(s_t; z_t)$, $r_t(s_t; z_t)$ and $w_t(s_t; z_t)$ such that:

(i) Given the prices, the value function $V_j^i(s_t; z_t)$ solves the recursive problem (9) of the representative type i households at age j, subject to the budget constraint (10) and time constraint (11). $c_{j,t}^i(s_t; z_t)$, $h_{j,t}^i(s_t; z_t)$, $\theta_{j,t}^i(s_t; z_t)$ and $\eta_{j,t}^i(s_t; z_t)$ are the associated policy functions for all generations and states;

(ii) The firm maximizes its profits in each period given prices, i.e. wages and rates of returns. In future versions of this paper, we will incorporate the intra-cohort heterogeneity, allowing different types of i, in the sense that workers have different wage levels. We have many ways to introduce intra-cohort heterogeneity;

(iii) All markets clear: Labour, capital and risk-free bond market clearing conditions are implied by (4), (3) and (2). These market clearing conditions and binding household budget constraints imply market clearing in consumption.

6 Parameterization

This section discusses parameter values for our baseline model with $J = 20$ and $i = 1$, i.e. each period represents 4 years and there is no intra-cohort heterogeneity. We have several parameters and constraints that are fixed and exogenous in our initial model, and we draw on the existing literature, as well as Canadian data, to set reasonable baseline values. Parameter values are summarized in Table 5 in Appendix A.

For the discount factor, we employ the standard 0.99 quarterly value, which is 0.8515 in our 4-year cohorts model. Capital's share is also standard in the literature at around 0.3. We follow Prescott (1986) and set the autocorrelation coefficient for TFP at a quarterly value of 0.95 (0.4401 for our model). The standard deviation of the error term in the TFP process (0.00763 quarterly, 0.0305 for our model) is also drawn from Prescott. We set the depreciation rate to be 0.048 quarterly (0.192 for our model).

Estimates of relative risk-aversion between one and two are common in the consumption literature, so we set $\gamma_c = 2$. Follow Heathcote et al. (2010), we set the reciprocal of the intertemporal elasticity of substitution for household's non-market time, γ_l, to 3.0. We calibrate the utility weight of leisure relative to market consumption, Ψ, to 21.833 such that the average hours worked in the market for households at YW, MW and W, estimated to be 30% of the time endowment H.

The following parameters we derive from the data. Survival probabilities, shown in Table 6 in Appendix A, are derived using Statistics Canada's 2009-2011 complete life table. We set the annual population growth rate at 1.2% (4.8% for our model), since Canadian population growth has fluctuated around 1% from the 1970s to the present and sat at 1.2% for 2012. Sales tax rates vary substantially across Canadian provinces, ranging from 5% to 15%. Using provincial population shares, we construct a weighted average tax that is about 12.3%[3]. To estimate average labour income tax, we use the 2011 Survey of Labour and Income Dynamics (SLID)[4]. For individuals aged 16-65, the total average income tax

[3]In addition to sales tax, Canada has additional consumption related taxes (e.g. on liquor, fuel and residence). OECD (2015) reports that taxes on income sources represent 47% of the tax burden, social security another 16%, taxes on goods and serves 25% and taxes on property 11%. Combining the latter two Canadians have a tax burden of 36%. If Canadians consume approximately 90% of their income, then these tax burden rates roughly match up with Canada's estimated average tax rates for income and consumption (77% of 16.7 is 12.9 of which 90% is 11.6, within range of 12.3), and a social security tax that represents approximately 1/3 of a 16.7% income tax is 5.5% (within range our 3.2%).

[4]The SLID is the primary source for income statistics in Canada. All estimates, from the SLID (as well as the General Social Survey), are weighted using survey weights.

paid is about \$7,000, dividing this amount by the average total income (approximately \$42,000) the effective tax is 16.7%. In the baseline parameterization, we assume all labour taxes go to social security. That is, the percentage of labour taxes that goes to pension, $ratio_s$, equals one. The effective tax rate on income for those aged 65-81 is 9.9%[5].

The SLID does not separately identify tax rates on various sources of income. So we cannot produce effective tax rates on earnings versus investment income. However, we do know that interest income is taxed identically to labour income, eligible dividends get a small tax break, capital gains are taxed on 50% of gains, and investments (up to a limit) in Tax Free Savings Accounts are not taxed at all. We begin with the assumption of the same tax rate for labour and investment income, and then do a sensitivity analysis with a two percentage point lower tax rate on investment income.

At this point, we assume that a household's productivity remains the same, at unit, over its life-cycle and do not take any experience effect into consideration. We leave the analysis of age-specified productivity to future study.

Finally, we have three time constraints that limit the amount of labour that workers can provide. First, we impose a time constraint on the semi-retired workers to reflect the large proportion retiring after age 65. The labour force participation rate of those age 66-81, as a fraction of the participation rate of those age 16-65, is just under 8%. Our oldest workers are therefore constrained against using 92% of their time.

Next, we consider that children and education both take up substantial amounts of time, limiting the hours available for labour market activities and leisure. We turn to the General Social Survey (GSS), cycle 19: time use (2005), to estimate the average hours spent on own-education activities, and on childcare. For the latter, we include time spent directly caring for children, as well as time spent on activities when childcare is a secondary activity. The age-specific constraints are given in Table 7 in the appendix.

7 Primary results

7.1 Baseline model

The life-cycle patterns of consumption, asset holdings and labour supply are depicted in Figure 1. Consumption is clearly hump shaped over the life cycle - a fact that has been well documented. The upper-right panel shows how households provide labour during life cycle.

[5]Income tax rates for seniors should be lower because of pension income splitting, age credits, and other tax credits.

In their earlier working ages, a household's labour supply critically depends on its time constraints of child rearing and education. This is because when the household is young, its leisure is quite stable and none of the time constraints are binding. When entering the semi-retired stage ($j > 12$), households work the maximum available units of time, i.e. $h_{j \in SR} = \iota_p H$. After retirement, the households do not work and enjoy all time as leisure. We also see the typical pattern of asset accumulation in working periods and decumulation in retirement (bottom-left panel), although the decumulation is steeper than observed in the data. This decumulation is problematic given that our focus is an exploration of the impact of population aging on asset prices, and retirement outcomes. To this end, we incorporate a bequest motive, and increasing health care costs, in subsequent sections of this paper.

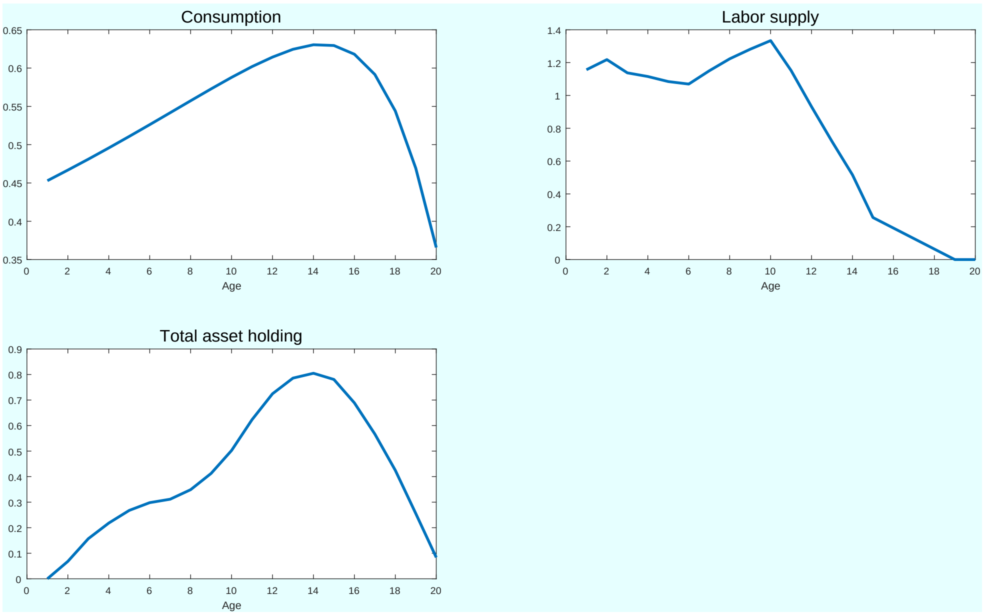

Figure 1: Lifecycle patterns of consumption, labour supply and total asset holding.

In order to understand and predict the impact of population aging on asset outcomes, it is important that our model closely reflects the observed portfolio allocation behaviour. Figure 2 shows the household's age-specific portfolio allocation. Similar to total asset holding, a household's captial holding also follows (roughly) a hump-shape pattern. The short sale on capital, i.e. negative capital holding, never happens and households invest in risky capital at all ages. On the other hand, the bond holding is negative among younger cohorts. On average, households sell bonds (borrow) in early ages and demand bonds in old ages. The curvature in the youngest cohorts is caused by household's time constraints on child rearing and education. In fact, relaxing these time constraints, the model exhibits a pattern of monotonically increasing bond demand with age. That is, the young borrow against their future labor income and insure the old by selling the bonds.

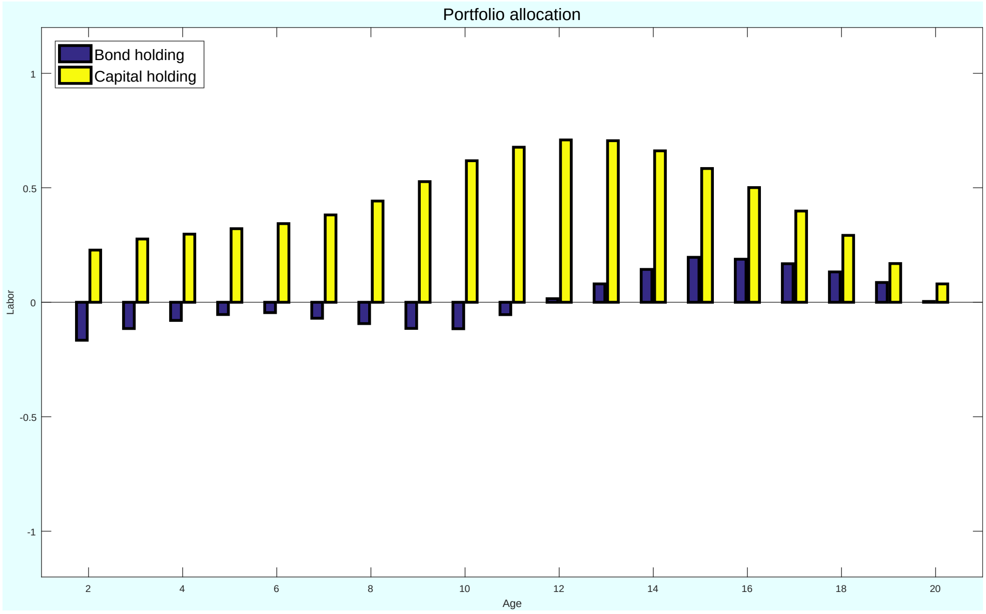

Figure 2: Lifecycle portfolio allocation under a two-pillar pension system.

Our results reconcile the empirical observation quite well qualitatively. Figure 3 is compiled from a representative survey of U.S. households portfolios where we count as risky assets any high risk is stocks, and any iras, mutual funds and other savings vehicles held in stocks. We also count as risky assets any non-stock iras, mutual funds and saving vehicles along with mixed funds, corporate bonds, other bonds, and money owed to respondent. Low risk assets are net of all debt and include checking accounts, t-bills, government and other savings bonds and certificates of deposit. On possible reason for the observed increase in assets among the oldest cohorts in Figure 3 is that households have bequest motives. Other possible reasons include increasing and uncertain health costs. Moreover, if longevity is greater for higher income households, we might expect survival bias to generate an uptick in assets for the oldest cohort. In order to explore these possibilities, we have implemented different versions of the model by considering bequest motives, introducing a constraint for health costs that increase with age. Our next step is to implement intra-cohort heterogeneity.

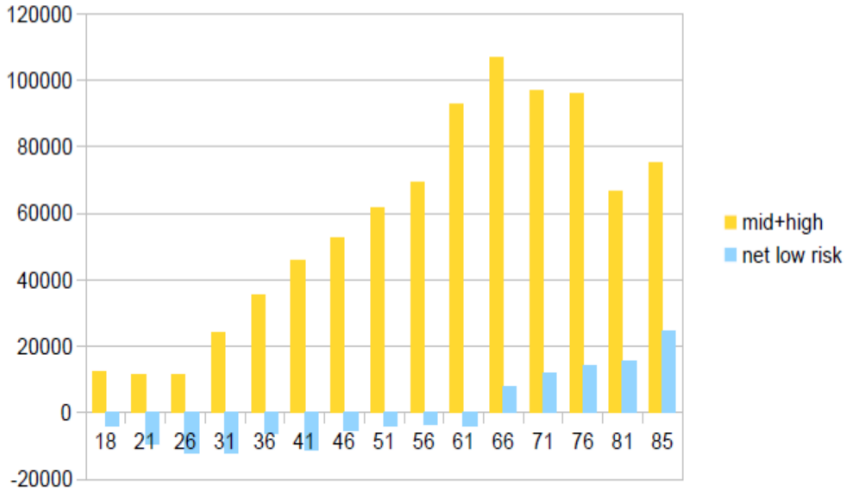

Figure 3: SCF summary: high + med risk versus low risk net of financial debt.

Table 1 provides the mean of asset returns in the baseline model with 2-pillar pension system. We note that expected capital returns and bond returns are very close, resulting in an equity premium of less than 1/2 a percent. The size of private pensions is just under 60% of GDP, which is close to what we observe empirically. Because we have no partially funded public pillar in this model, the public holdings of risky assets and the funded tax rate are both zero.

Table 1: Variable mean values: two-pillar pension model.

Variable	Mean
Expected capital return $(E_t(r_{t+1}))$	0.3052
Bonds return $(\bar{r}_t)$	0.3012
Equity premium $(E_t(r_{t+1}) - \bar{r}_t)$	0.0040
Public holdings risky assets/GDP	N/A
Private holdings risky assets/GDP	0.5964
Private holding bonds/GDP	0.0000
$\tau^G_{s,t}$	N/A
$c^i_{20,t}$	0.4528

7.2 Impact of population structure on portfolio allocation and asset returns

Next we examine the impact of demographic factors on economic activity. At this point, we focus on how population aging affects asset returns. To do so, we conduct counterfactual simulations in which population aging varies, in the sense of changing survival probability for old households. Let $\triangle^{O}$ denote the percentage change in survival probabilities of households over 65 years old[6]. Simulation outcomes are listed in Table 2. Portfolio allocation changes are presented in Figure 4.

Table 2: Means with various $\triangle^{O}$: two-pillar pension model

$\triangle^{\mathbf{O}}$	**Dep Ratio**	$E_t\left(r_{t+1}\right)$	$\bar{r}_t$	$E_t\left(r_{t+1}\right)-\bar{r}_t$	Risky asset holdings over GDP (K_t/Y_t)
-4%	0.2724	0.3182	0.3180	0.003979	0.5928
-2%	0.2826	0.3119	0.3318	0.003994	0.6004
Baseline	0.2933	0.3052	0.3012	0.004006	0.6084
+2%	0.3043	0.2991	0.2987	0.004015	0.6168
+4%	0.3156	0.2925	0.2921	0.004020	0.6256

The model predicts a clear relationship between population aging and asset returns: The higher the survival rate of the old, the lower the returns. Two reasons may explain why increased longevity results in lower asset returns. One reason is that increased longevity influences household's precautionary motive. Households expecting to survive longer have an added incentive to save so as to insure themselves against outliving their assets. This results in larger capital stock and lower asset returns. The other reason is the scarcity of labour relative to capital. In our model, there is no child generation, thus, higher values of $\triangle^{O}$ imply a higher old age dependency ratio. A larger share of dependents is associated with a lower proportion of labour available for the firm, and therefore decreases the return on capital. Moreover, not surprisingly, the more aging the population is, the higher ratio of risky asset holdings relative to GDP.

[6]In this experiment, each generation over 65 years old has the same $\triangle^{O}$. A more sophisticated study may specify different survival rate changes for each old cohort.

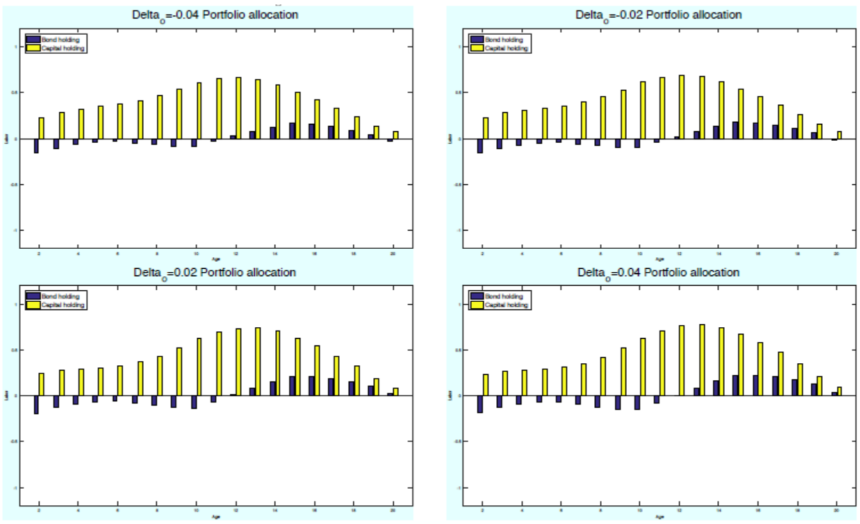

Figure 4: Portfolio allocation with various Δ_O: two-pillar pension model.

7.3 Three pillar pension model

Canada has a three pillar federal pension system. The first pillar is a universal benefit (Old Age Security, Guaranteed Income Supplement, and spousal allowance), the second pillar is an earnings-related contributory program (Canada/Quebec Pension Plan, which has similarities to the U.S. social security system), and the third pillar is private retirement savings (which include individual savings as well as employer plans). Our two pillar model includes the first pillar and private savings. Our three pillar model incorporates an earnings-related contributory program.

The predictions of this model are presented below. Assuming a 20% income replacement ratio for the employment-related pension plan, we note that the introduction of the earnings-related retirement program results in an increase in the magnitude of total retirement savings. Now private investment in risky assets constitute about 56% of GDP, and the earnings-related pillar comprises just under 10% of GDP. The equity premium is increased little in this model, and the mean returns to our risky and risk free assets are down. Note that consumption, for the oldest cohort, has increased (from 0.45 to 0.49) going from the two to the three pillar model

Figure 5 shows the lifecycle portfolio allocation under a three pillar pension system. There is a slight shift in the asset holdings for younger generations, but the lifecycle pattern remains substantively similar.

A key concern for policy makers is the adequacy of retirement income for boomers entering retirement. Another major concern is the impact on the economy of supporting an aging population. We again change the survival rates for cohorts over age 65 and find that the impact on the economy is small. The portfolio allocation changes moderately (see Figure 6), the equity premium increases slightly (less than 3%), and the average returns to our risky and risk free assets fall. In terms of the size of the pension system, as expected, with an increase in survival rates, the ratio of private saving to GDP rises about 2% for an 2% increase in Δ^O.

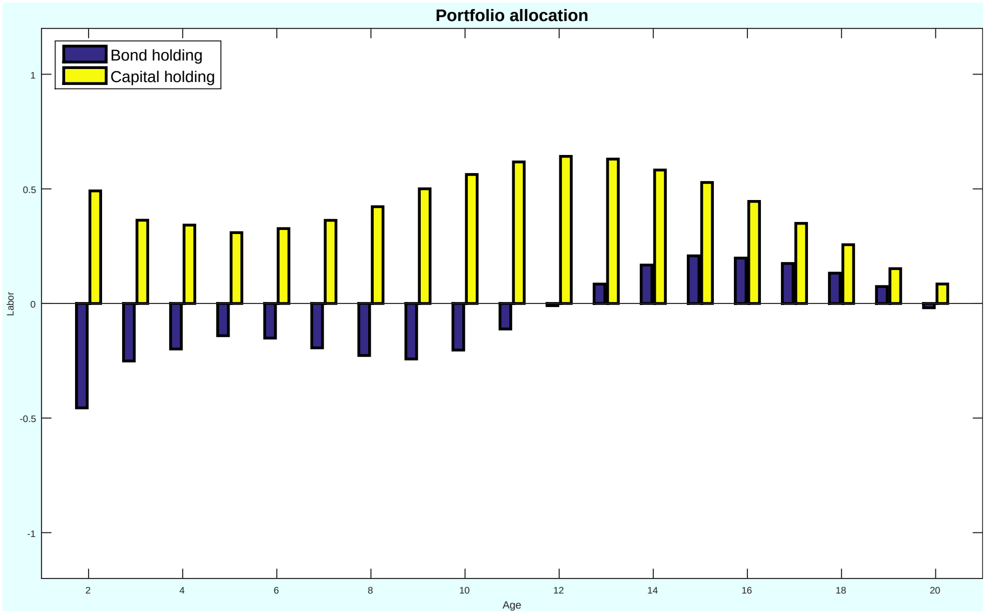

Figure 5: Lifecycle portfolio allocation under a three-pillar pension system.

Table 3: Variable mean values: three-pillar pension model.

Variable	**Mean**
Expected capital return $(E_t(r_{t+1}))$	0.2826
Bonds return $(\bar{r}_t)$	0.2780
Equity premium $(E_t(r_{t+1}) - \bar{r}_t)$	0.0046
Public holdings risky assets/GDP	0.0968
Private holdings risky assets/GDP	0.5581
Private holding bonds/GDP	-0.0645
$\tau^G_{s,t}$	0.0300
$c^i_{20,t}$	0.4918

Table 4: Means with various $\triangle^O$: three-pillar pension model

$\triangle^{\mathbf{O}}$	**Dep Ratio**	$E_t(r_{t+1})$	$\bar{r}_t$	$E_t(r_{t+1}) - \bar{r}_t$	Risky asset holdings over GDP (K_t/Y_t)
-4%	0.2724	0.2960	0.2914	0.0043	0.6287
-2%	0.2826	0.2895	0.2848	0.0044	0.6373
Baseline	0.2933	0.2826	0.2780	0.0046	0.6464
+2%	0.3043	0.2759	0.2710	0.0047	0.6561
+4%	0.3156	0.2687	0.2637	0.0048	0.6666

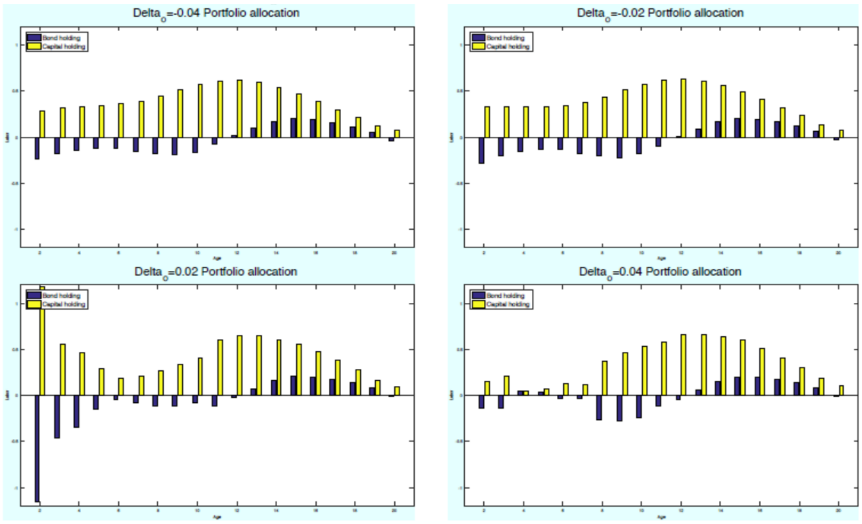

Figure 6: Portfolio allocation with various Δ^O: three-pillar pension model.

7.4 Further modifications: bequest and health care costs

As we note earlier, we aim to match asset holding among the older cohorts, and that the decumulation among the oldest groups is too steep. As such we consider the addition of a simple bequest motive, and of health care costs that increase linearly for the oldest cohorts.

With a simple and small bequest motive, we note that households do retain more assets in older ages, but the lifecycle pattern of portfolio allocation can change dramatically.

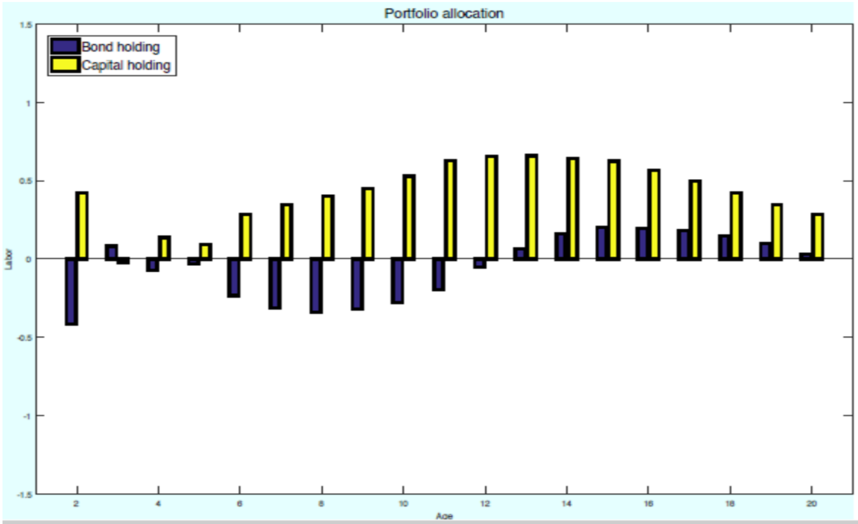

Figure 7: Lifecycle portfolio allocation in baseline model with bequest motive.

Health care costs have little impact on the decumulation problem for risky assets, but do slow the decumulation of bonds.

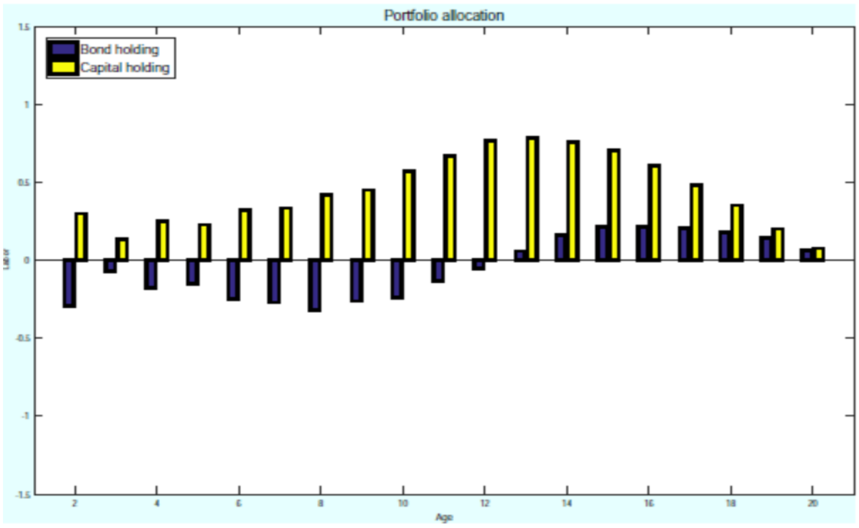

Figure 8: Lifecycle portfolio allocation in baseline model with linearly increasing health costs for retirees.

Our next step is to consider intra-cohort heterogeneity as we expect that differences in survival rates could generate a reduced decumulation rate. Moreover, we expect that retirement outcomes will vary broadly across different demographic groups, and policy makers should be aware of the implications of demographic structure across the population, not just on aggregate.

8 Conclusion and future research

With the large baby-boom cohort entering retirement, many are concerned that the expected drop in saving and investment will result in substantially diminished asset prices and compromised pension plans. This paper developes a large scale computable Overlapping Generations model to quantify the impact of population structure on asset values. Results from our counterfactual excersises suggest that asset prices are moderately lower with an older population. Specifically, a 4% increase in the survival probability of households over age 65 results in a 4.16% drop in the return on capital, and a 3.02% drop in the return on bonds.

This paper makes three specific contributions to the literature. First, we investigate the impact on asset prices of a higher old age dependency ratio in a model that incorporates both a risk-free and a risky asset, with endogenous returns. Second, we test the implications of demographic structure under a three-pillar pension system which includes a publicly administered, partially funded, employment-related pension. This framework can also be used to assess the implications of the proposed expansion of the Canada Pension Plan. Finally, we generate age-specific portfolio allocations that are consistent with the data, with the exception of the oldest cohort. We explore additional mechanisms to address this concern (e.g. bequest motive, health care costs and, as a next step, intra-cohort heterogeneity). Both the bequest motive and health care costs modestly improve the asset holdings at the oldest cohort, but not to the extent depicted in the data. Our next step is consideration of intra-cohort heterogeneity with productivity and longevity differences.

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A Parameter values

Table 5: ***Parameters List: Baseline Model***

Parameter	Value	Description
J	20	Each period represents 4 years
I	1	No intra-cohort heterogeneity
H	4.0	Available time to spend for households
β	0.8515	Discount factor
α	0.3	Capital share of production
ρ_z	0.4401	Autocorrelation coefficient for the TFP process
σ_z	0.0305	The standard deviation of error term in the TFP process
ρ_q	0.4401	Autocorrelation coefficient for the TFP process
σ_q	0.0305	The standard deviation of error term in the TFP process
δ	0.192	Depreciation rate
n	0.0489	Population growth rate
γ_c	2.0	Relative risk-aversion
γ_l	3.0	Reciprocal of the intertemporal elasticity of substitution for household's non-market time
Ψ	21.833	Utility weight of non-market time relative to market consumption
τ_c	0.123	Consumption tax rate
τ_r	0.167	Tax on investment income
$\tau_h + \tau_s$	0.167	Labor income tax
$ratio_s$	1.0	Percentage of labour tax to pension (Social security deduction)
τ_p	0.099	Tax on pension income
ι_p	0.08	Labour time constraint at stage SR, % of H
ε_j	1.0	Age-specific productivity (efficient labour) profile
χ	1.0	proportion of type i households within a generation.

Table 6: ***Survival Probabilities from age*** j ***to*** $j+1$***:*** ϕ_j

	Age (year)	**Model Age**	**Survival Prob.**
Young-working (YW)	18 - 21	1	0.9982
	22 - 25	2	0.9979
	26 - 29	3	0.9980
	30 - 33	4	0.9977
Middle-working (MW)	34 - 37	5	0.9971
	38 - 41	6	0.9961
	42 - 45	7	0.9947
	46 - 49	8	0.9927
Mature-working (W)	50 - 53	9	0.9895
	54 - 57	10	0.9849
	58 - 61	11	0.9781
	62 - 65	12	0.9681
Old-working (SR)	66 - 69	13	0.9533
	70 - 73	14	0.9316
	74 - 77	15	0.8995
	78 - 81	16	0.8527
Retirement (R)	82 - 85	17	0.7848
	86 - 89	18	0.6887
	90 - 93	19	0.5589
	94 - 97	20	0.0000

Table 7: ***Time on Child Rearing and Education at age*** j***:*** FC_j ***and*** FE_j

	Age (year)	**Model Age**	FC_j (% of H)	FE_j (% of H)
Young-working (YW)	18 - 21	1	0.00	0.16
	22 - 25	2	0.03	0.11
	26 - 29	3	0.07	0.03
	30 - 33	4	0.14	0.02
Middle-working (MW)	34 - 37	5	0.14	0.01
	38 - 41	6	0.12	0.01
	42 - 45	7	0.11	0.01
	46 - 49	8	0.05	0.01

B Algorithm

B.1 Steady state

We first consider a steady state in which productivity is constant and normalized to unity: $z_t = 1$ for all t. In this setting, all normalized quantities and values are constant and their steady-state values are indicated with an asterix. The steady state conditions are listed below.

$$c_{j+1}^{i*} = \left(\beta\phi_j^i\left(1+\left(1-\tau_r\right)r^*\right)\right)^{\frac{1}{\gamma_c}} c_j^{i*} \tag{24}$$

$$\bar{r}^* = r^* \tag{25}$$

$$h_j^{i*} = H_j - \left(\frac{1}{\Psi^i}\left(\frac{1-\tau_h-\tau_s}{1+\tau_c} w^*\varepsilon_j^i\left(c_j^{i*}\right)^{-\gamma_c} - \lambda_j^{2*}\right)\right)^{-\frac{1}{\gamma_h}} \tag{26}$$

$$r^* = \alpha\left(\frac{\sum_j\sum_i\theta_j^{i*}N_j^i}{\sum_j\sum_i\varepsilon_j^i h_j^{i*}N_j^i}\right)^{\alpha-1} - \delta \tag{27}$$

$$w^* = (1-\alpha)\left(\frac{\sum_j\sum_i\theta_j^{i*}N_j^i}{\sum_j\sum_i\varepsilon_j^i h_j^{i*}N_j^i}\right)^{\alpha} \tag{28}$$

$$\theta_j^{i*} = \left\{\begin{array}{c}\left(1-\tau_h-\tau_s\right)w^*\varepsilon_j^i h_j^{i*} + \left(1+\left(1-\tau_r\right)r^*\right)\theta_{j-1}^{i*} \\ +\left(1-Gauss-Seidelalgorithm\tau_p\right)p_j^* + \xi^* - c_j^{i*}\end{array}\right\} \tag{29}$$

$$p^* = \frac{\tau_s w^*\sum_j\sum_i\varepsilon_j^i h_j^{i*}N_j^i}{\sum_{j=4}^J\sum_i N_j^i} \tag{30}$$

$$\xi^* = \frac{\sum_j\sum_i\left(1-\phi_{j-1}^i\right)\left(1+\left(1-\tau_r\right)r^*\right)\theta_{j-1}^{i*}N_{j-1}^i}{(1+n)\sum_j\sum_i N_{j,t}^i} \tag{31}$$

$$c_J^{i*} = \left(1+\left(1-\tau_r\right)r^*\right)\theta_{j-1}^{i*} + \left(1-\tau_p\right)B_j^* + \xi^* \tag{32}$$

$$0 = \lambda_j^{2*}\left(H_j^c - h_j^{i*}\right) \tag{33}$$

In the steady state, there is no risk associated with capital. This implies that the returns on risk-free bonds and risky assets are the same ($\bar{r}* = r^*$) and η_j^{i*} is undetermined. We assume $\eta_j^{i*} = 0$ for all (j,i), i.e. only capital is traded in the steady state. Equation (33) is the complementary slackness of the time constraint. Note the constant population growth rate n only shows up in (31), which determines steady state accidental bequests. Even though n raises both aggregate capital $K^*$and labour H^*, it does so at the same pace such that the ratio of capital to labor $\frac{K^*}{H^*}$ remains constant in the steady state, as shown in (27) and (28).

To solve the steady state, we starts by making an initial guess of accidental bequest ξ^*. Next, we guess a value of return on capital r^* and compute corresponding value of w^*. Given the prices, the steady state consumption c_j^{i*} and labour supply h_j^{i*} can be derived. Next, r^* and ξ^* are updated until convergence. The detail algorithm is as follows.

1. Make an initial guess of accidental bequest $\xi^{*(0)}$,

   (a) Make an initial guess of capital return $r^{*(0)}$,

   (b) Compute w^* using (27) and (28),

      i. Make an initial guess of all types' consumption at age 1, $c_1^{i*(0)}$,

      ii. Compute c_j^{i*} $(j = 2, \ldots, J-1)$ using (24) and h_j^{i*} $(j = 1, \ldots, R)$ using (26) and (33),

      iii. Compute p^* using (30) and θ_j^{i*} using (29),

      iv. Compute c_5^{i*} using (32),

      v. Find $c_1^{i*(q)}$ backwardly using (24),

      vi. Check for convergence of c_j^{i*}: If

$$\left| \frac{c_1^{i*(q)}}{c_1^{i*(q-1)}} - 1 \right| < \epsilon,$$

      go to the next step. Otherwise, for each type i, update the age 1 consumption as $c^{i*(q+1)} = (1 - \zeta_c)\, c_1^{i*(q-1)} + \zeta_c c_1^{i*(q)}$ and return to step (ii). ζ_c is the damping parameter that controls the process of convergence for c_j^{i*}. We use $\epsilon \in [10^{-9}, 10^{-6}]$.

   (c) Use values of h_j^{i*} and θ_j^{i*} to compute $r^{*(p)}$ following equation (27),

   (d) Check for convergence of r^*: If

$$\left| \frac{r^{*(p)}}{r^{*(p-1)}} - 1 \right| < \epsilon,$$

   go to the next step. Otherwise, update return on capital as $r^{*(p+1)} = (1 - \zeta_r)\, r^{*(p-1)} + \zeta_r r^{*(p)}$ and return to step (b). ζ_r is the damping parameter that controls the process of convergence for r^*.

2. Compute $\xi^{*(p)}$ using (31),

3. Check for convergence of ξ^*: If

$$\left| \frac{\xi^{*(p)}}{\xi^{*(p-1)}} - 1 \right| < \epsilon,$$

end. Otherwise, update accidental bequest as $\xi^{*(p+1)} = (1 - \zeta_x)\, \xi^{*(p-1)} + \zeta_x \xi^{*(p)}$ and return to step (a). ζ_x is the damping parameter that controls the process of convergence for ξ^*.

B.2 Dynamics

To solve the dynamics of the model, we extend the algorithm of Hasanhodzic and Kotlikoff (2015) to incorporate endogenous labour supply. The algorithm consists of two loops. In the outer loop, consumption functions of each representative households are solved by applying Judd et al. (2009, 2011) generalized stochastic simulation algorithm (GSSA) to our OLG model. In the inner loop, using a combination of numerical techniques, we compute each agent's labour supply, asset holdings and the risk-free rate that clears the safe bond market. A more detailed overview of the algorithm is descried in the following subsections.

One important difference between our computation and Hasanhodzic and Kotlikoff (2015) is the definition of state variables. They use the vector of household cash-on-hand as state variables, which are defined as the total available resources for consumption and investment, including labour income and return on assets, net of pension payments. Note household cash-on-hand itself is a function of current prices: what is predetermined is households' asset positions at the end of last period, but we need current wage rate and asset returns to compute current cash-on-hand. If labour supply is exogenously given, as in Hasanhodzic and Kotlikoff (2015), because the aggregate capital is pre-determined in the previous period, the wage rate w_t and return on capital r_t in period t are both "known" once the period t productivity z_t is realized. Therefore, it is straightforward to compute cash-on-hand, in particular when using GSSA in which the path of z_t is "pre-determined".

However, when households choose labour supply endogenously, as in our paper, the computation of cash-on-hand is more complex. This means that we have to solve a non-linear fixed point problem in order to compute current states from past states and current decisions. To bypass the complexity, in our algorithm, we exclude labour incomes and only use household net worth - the market value of all assets - as state variables. This avoids solving the fixed point problem for each representative household's labour supply and only focuses on the aggregate labour supply, which determines current return on capital r_t and in turn the net worth x_t. To solve r_t, a guess-and-verify method is applied.

The following is the step-by-step description of the algorithm.

B.2.1 Initialization

1. The starting point of the simulation is the steady state values of state variables: $(s_0; z_0) = \left(x_2^{1*}, \dots x_j^{i*}, \dots x_J^{I*}; z^* = 1\right)$.

2. Approximate $(J-1) \times I$ consumption functions by degree 1 polynomials in the variables with the following initial guess of coefficients:

$$\begin{pmatrix} c_1^1 \\ \vdots \\ c_j^i \\ \vdots \\ c_{J-1}^I \end{pmatrix} = \begin{pmatrix} 0 & \frac{0.5c_1^{1*}}{x_2^{1*}} & \dots & 0 & \dots & 0.5c_1^{1*} \\ 0 & 0 & \frac{0.5c_1^{2*}}{x_2^{2*}} & 0 & \dots & 0.5c_1^{2*} \\ \vdots & 0 & \dots & \frac{0.5c_j^{i*}}{x_{j+1}^{i*}} & \dots & 0.5c_j^{i*} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & \dots & \frac{0.5c_{J-1}^{I*}}{x_J^{I*}} & 0.5c_{J-1}^{I*} \end{pmatrix} \begin{pmatrix} 1 \\ x_2^1 \\ \vdots \\ x_j^i \\ \vdots \\ x_J^I \\ z \end{pmatrix}$$

   Denote the coefficient matrix as $B^{(0)} = \left(b_1^{1(0)}, \dots, b_j^{i(0)}, \dots b_{J-1}^{I(0)}\right)^T$. The purpose of this algorithm is to find the convergent matrix B, given a simulated path of productivity z_t. Note $\left(c_1^{1*}, \dots, c_j^{i*}, \dots c_{J-1}^{I*}\right)^T = B^{(0)} (s_0; z_0)^T$.

3. Take draws of the exogenous path of the shock ε_t for T periods. We set T to $10 \times (J-1) \times I$, which corresponds to 10 observations per polynomial coefficient. Simulate and save the path for productivity z_t for the following computation.

B.2.2 Iteration loops

1. Given the intial guess of $B^{(p-1)}$ and productivity path $\{z_t\}_{t=1}^T$, simulate the model forward to solve the state space for $t = 1, \dots T$. We proceed as follows for each period t:

   (a) Find the return on capital $r_t^{(p-1)}$ that clears the labour market. Guess an initial current return on capital $r_t^{(0)}$.

i. Compute the wage rate as:

$$w_t^{(0)} = (1-\alpha)\, z_t \left(\frac{r_t^{(0)} + \delta}{\alpha z_t} \right)^{\frac{\alpha}{1-\alpha}}$$

ii. Given total asset holdings from the previous period $\theta^i_{j-1,t-1}$ and corresponding share of bonds $\eta^i_{j-1,t-1}$, we can compute the state space for period t, $s_t = \left(x^1_{2,t-1}, \ldots x^i_{j,t-1}, \ldots x^I_{J-1,t-1}\right)$ following (8).

iii. Using the state vector $(s_t; z_t)$, for each representative household, to compute its consumption $c^{i(q-1)}_{j,t}$given current guess $B^{(p)}$. Note the superscript p and q denote the current iteration on B and r_t, respectively.

iv. Given $w_t^{(q-1)}$ and $c^{i(q-1)}_{j,t}$ derived above, derive $h^{i(q-1)}_{j,t}$ using (20) and (21).

v. Compute aggregate capital and labour supply using (3) and (4).

vi. Using (22) to get $r_t^{(q)}$. Check convergence if

$$\left| \frac{r_t^{(q)}}{r_t^{(q-1)}} - 1 \right| < \epsilon,$$

save corresponding values as $c^{i(p-1)}_{j,t}$, $h^{i(p-1)}_{j,t}$, $r_t^{(p-1)}$, $w_t^{(p-1)}$, compute $p_t^{(p-1)}$using (5) and go to the next step. Otherwise, update return on capital as $r_t^{(q+1)} = (1-\zeta_r)\, r_t^{(q-1)} + \zeta_r r_t^{(q)}$ and return to step (i).

(b) Compute type- and age-specific total asset holding, $\theta^{i(p-1)}_{j,t}$, using (10). Note that the sum of asset holdings of households of age 1 through $J-1$ equals the capital stock at the beginning of period $t+1$.

(c) Next, we need to compute choices of share of risk-free bonds, $\eta^{i(p-1)}_{j,t}$, for households with age 1 to $J-1$, as well as the risk-free rate $\bar{r}_t^{(p-1)}$ that clears the bond market. Recall that these are needed to compute the state variables variables in period $t+1$. To do so, we first construct a vector function $F\left(\bar{r}_t; \theta^{i(p)}_{j,t}, z_{t+1}, B^{(p)}\right)$ to compute the net supply of risk-free bonds $\sum_j \sum_i \eta^i_{j,t}\theta^i_{j,t}N^i_{j,t}$ given $\bar{r}_t$. Then, the market clearing risk-free rate $\bar{r}_t^{(p-1)}$ is solved when $F(\cdot;\cdot) = 0$.

i. Given $\bar{r}_t^{(q)}$, solve $\eta^{i(q)}_{j,t}$ using (19) and budget constraints.

A. The expectation is approximated by 4-nodes Gaussian quadrature.

B. To approximate each $c^{i(q)}_{j+1,t+1}$ on the right hand side of equation (19), we need to compute the entire distribution of household net worth, which depends on all households' η's, $\eta^{i(q)}_{1,t}, \ldots, \eta^{I(q)}_{J-1,t}$. To solve this non-linear system of $(J-1)\times I$ equations with $(J-1)\times I$ unknowns, we use the Gauss-Seidel algorithm, which reduces the problem of solving for $(J-1)\times I$ unknowns simultaneously in $(J-1)\times I$ equations to that of iteratively solving $(J-1)\times I$ equations in one unknown. Newton's method is used to solve each of these non-linear equations in one unknown $\eta_{j,t}^{i(q)}$. Notice that corresponding returns on capital are computed using similar guess-and-verify process as step (a) above.

ii. Using $\eta_{j,t}^{i(q)}$ found above to compute the value of $F\left(\bar{r}_t^{(q)};\theta_{j,t}^{i(p)},z_{t+1},B^{(p)}\right)$.

iii. Apply Matlab routine *"fzero"* to find $\bar{r}_t^{(p-1)}$ that clears the risk-free bond market.

(d) Using $\bar{r}_t^{(p-1)}$, $\theta_{j,t}^{i(p-1)}$and $\eta_{j,t}^{i(p-1)}$, we can construct $x_{j+1,t+1}^{i(p-1)}$ and return to step (a) for period $t+1$.

2. For period t with state $(s_t;z_t)$, equation (18) implies

$$c_{j,t}^{i(p)}\left(s_t;z_t\right)=\left(\beta\phi_j^i E_t\left[\left(1+r_{t+1}^{(p-1)}\left(s_{t+1};z_{t+1}\right)-\tau_r\right)\left(c_{j+1,t+1}^{i(p-1)}\left(s_{t+1};z_{t+1}\right)\right)^{-\sigma_c}\right]\right)^{-\frac{1}{\sigma_c}},$$

where $r_{t+1}^{(p-1)}$ and $c_{j+1,t+1}^{i(p-1)}$ are computed use $\left\{\bar{r}_t^{(p-1)},\theta_{j,t}^{i(p-1)},\eta_{j,t}^{i(p-1)},z_t\right\}_{t=1}^{T}$ derived in step 1. The expectation is again evaluated using 4-nodes Gaussian quadrature.

3. For each representative household, regress $c_{j,t}\left(s_t;z_t\right)$ on $(1,s_t,z_t)$ using regularized least squares with Trikhonov regularization. Denote the estimated coefficients as $b_j^{i(p)}$. Combine all $b_j^{i(p-1)}$ together, we get $B^{(p)}$.

4. Check convergence: If

$$\frac{1}{(J-1)\,I}\sum_{j=1}^{J-1}\sum_{i}\sum_{t=1}^{T}\left|\frac{x_{j,t}^{i(p)}}{x_{j,t}^{i(p-1)}}-1\right|<\epsilon,$$

end. Otherwise, update $B^{(p+1)}=\left(1-\zeta_b\right)B^{(p-1)}+\zeta_b B^{(p)}$. ζ_b is the damping parameter that controls the process of convergence for B.

C Business Cycle Statistics

Because variables are in a four-years frequency, we focus on the relative standard deviation (RSD) of a macro variable relative to that of the output. Qualitatively consistent with stylized facts, consumption is much less variable than output and investment and asset returns are much more variable. However, the model fails to replicates volatility of other variables. For example, in real data, real wage and capital stock are both much less volatile than output, while hours worked are as volatile as output. Although the model correctly predict the co-movements of most variables, its prediction on bond returns and labour supply are in wrong direction. Moreover, real wage is more likely to be acyclical in real data rather than strong procyclical as predicted by the model. However, these are common failures in this type of model.

Table 8: Variable Statistics: two-pillar pension model.

Variable	Steady S.	Mean	RSD (%)	RSD /RSD(Y)	Corr(x,Y)
Output (Y_t)	14.6595	14.6273	5.6400	1.0000	1.0000
Consumption (C_t)	10.9597	10.9625	5.1101	0.9061	0.7610
Capital (K_t)	8.9184	9.0547	17.1683	3.0440	0.7626
Wage (w_t)	0.5657	0.5646	6.9956	1.2404	0.9326
Capital return (r_t)	0.3011	0.3056	38.9743	6.9104	0.0892
Expected capital return ($E_t(r_{t+1})$)	–	0.3052	29.4872	5.2283	0.3820
Bonds return ($\bar{r}_t$)	–	0.3012	29.8196	5.2872	0.2192
Equity premium ($E_t(r_{t+1}) - \bar{r}_t$)	–	0.0040	–	–	0.1429
Labour Supply (L_t)	18.1391	18.1583	2.6708	0.4736	-0.3456
Investment (I_t)	1.7123	1.6710	33.7678	5.9872	0.4908
Variability of lifetime consumption (vol_t^{TC})	0.0745	0.0754	12.5892	2.2321	-0.2374
Risky asset holdings over GDP (K_t/Y_t)	0.6084	0.6164	13.5737	2.4067	0.5644

Figure 9 displays the evolution of the return on safe bonds, the return on capital, the wage and the equity premium. The premium is defined as the expected return on capital less the return on bonds in period t. Although stock returns are more volatile than bond returns, on average the returns are very close. That is, the equity premium is very small. The annualized premium is only 0.01%. Even with a higher risk aversion parameter - $\gamma_c = 10$, the premium remains small, at 0.08% per year. Thus, we do not explain the equity puzzle in this model. This type of model - OLG models with aggregate uncertainty and many cohorts - usually fails to generate comparable risk-premium. For example, Reiter (2015) finds the annualized 0.066% risk premium when the degree of risk aversion γ_c equals 2 and 0.182% when $\gamma_c = 6$. Hasanhodzic and Kotlikoff (2013) also predict very small risk premium. However, when a disaster based depreciation shock is introduced to their framework, Hasanhodzic and Kotlikoff (2015) generate a risk premium close to the equity premium observed in the stock market.

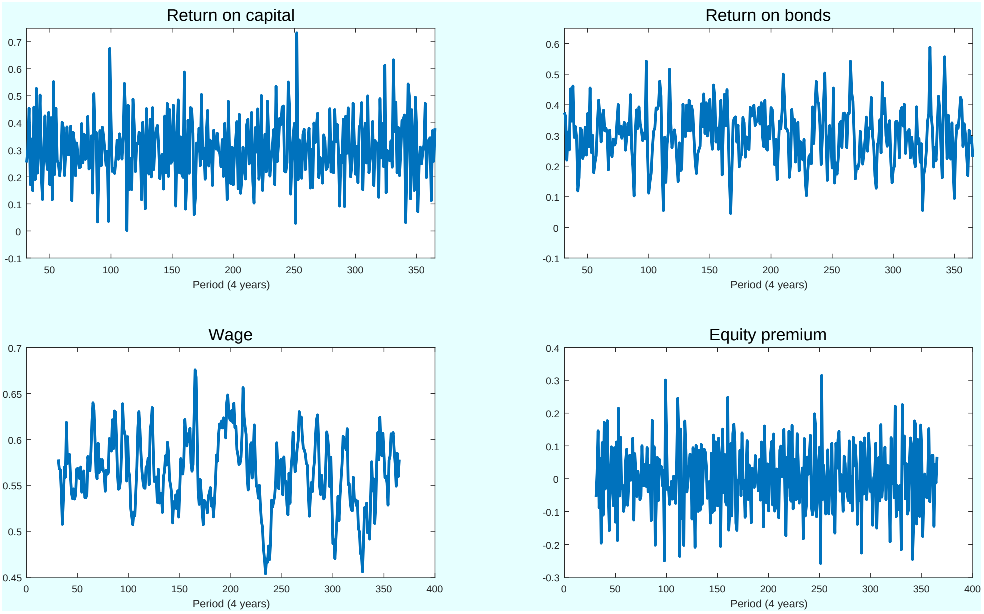

Figure 9: Evolutions of prices: two-pillar pension model.