Assessing economic welfare. Do behavioral and social factors change the picture?

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Abstract

This study empirically investigates economic inequality and poverty taking account of behavioral and social factors that may determine an individual's valuation of income (individual welfare). Using survey data on income satisfaction, we estimate an extended utility function of income that allows for reference-dependence, loss aversion, diminishing sensitivity for gains and losses, and preferences about inequality. Our results show that reference-dependence and loss aversion affect welfare, particularly in the lower part of the income distribution. Hence, reference-dependence appears to change our picture of poverty but barely our picture of income inequality. Generally, inequality indices and poverty measures that take account of behavioral and social factors may provide supplementary information to conventionally calculated inequality measures.

Keywords: inequality, equivalent income, reference-dependent preferences, welfare, marginal utility of income

JEL Classification: I32

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1 Introduction

Economic inequality is among the most pressing social issues globally. Since policies, such as income taxation, often directly impact on the income distribution, policy-makers and the general public have a need for indicators of economic inequality. "Good" indicators are therefore a prerequisite for effective policy-making and policy evaluation. A variety of inequality indices exist for summarizing information about the income distribution (Jenkins and Van Kerm 2012). These indices are usually calculated from micro data on household income, and equivalence scales are employed to adjust for differences in household composition and size. As an example, the OECD uses Gini coefficients from equivalized household incomes to document the trends and recent developments in income inequality (OECD 2015).

"Good" indicators of economic inequality are expected to measure aspects of the income distribution that determine economic living conditions. This implies that economic inequality should not be assessed against the perception of the general public. However, Jäntti et al. (2014) point out that views of economic inequality, particularly with respect of welfare consequences of policy changes, are often divergent between official organizations and the general public. An explanation for disagreeing views may be that standard poverty and welfare measurement is based on models of economic agents that do not integrate insights from behavioral economics and social preferences, such as reference-dependence and inequality aversion. In consequence, the conventional calculation of inequality indices does not capture behavioral and social factors that may determine people's perceptions of economic inequality. For instance, we have no clear understanding of how loss aversion and inequality aversion affect the (marginal) utility of income and equivalent income and, in turn, our picture of inequality.

This study empirically investigates the properties of the (marginal) utility of income using survey data on income satisfaction. We make the following novel contributions to the literature. First, we estimate an individual utility function of income by incorporating reference-dependence

and preferences about inequality. A particular motivation for taking account of preferences about income inequality is that the income distribution represents a frame of reference for comparison purposes. Frank (1997) interprets such a frame of reference as a public good. Second, we provide an estimate of the equivalence scale elasticity based on a richer parameterization of the utility function compared to specifications used in previous research. Third, we are able to calculate an alternative equivalent income, as suggested by Jäntti et al. (2014), that may be used for welfare measurement. Finally, we scrutinize how our picture of economic inequality is affected when we use inequality indices that take account of reference-dependence and preferences about inequality.

Using data from the Socio-Economic Panel Study for the years 2002 to 2014, we carry out a grid search procedure to estimate a nonlinear-in-parameters utility function. The grid search approach is a flexible tool that, in our case, allows to control for unobserved individual fixed effects. Our estimates of the equivalence scale elasticity parameter are in a narrow range between 0.4. and 0.5, which is of comparable magnitude to other studies. We further estimate the elasticity of the marginal utility of income to be about 1.2. Reference-dependence and loss aversion affect welfare, particularly in the lower part of the income distribution. Hence, reference-dependence appears to change our picture of poverty but barely our picture of income inequality.

2 Conceptual framework

Inequality indices used to describe the extent of economic inequality generally summarize the distribution of household incomes or household consumption expenditures, which are the variables most commonly studied (Jenkins and Van Kerm 2012). Incomes are, however, not readily comparable across households due to different household sizes, households compositions, and economies of scale. Therefore, equivalence scales are regularly employed in poverty and inequality calculations to allow for benefits from shared consumption (for a discussion, see, e.g.,

Jenkins and Cowell 1994). As a result, equivalent incomes are of great importance to inform the public debate about the extent of economic inequality.

Governments and NGOs communicate inequality indices that are primarily based on equivalent incomes that are calculated using so-called expert scales. An example is the OECD scale that assigns a weight of 1 to the household head, of 0.5 to further adults, and of 0.3 to children under 15 years of age.[1] As an alternative to expert scales, equivalent scales can be estimated empirically using data on consumption or expenditure data (see, e.g., Nelson 1992, Pendakur 1999) or survey data on income satisfaction (Schwarze 2003).

Despite of recent advances in research on the estimation of the equivalence scale elasticity (e.g., Vendrik and Woltjer 2007, Layard et al. 2008)—in particular by accessing and using new data sources such as subjective data on income satisfaction—, current research lacks an understanding about whether and how behavioral and social factors determine the utility of income. Knowledge about the properties of the individual utility function are important because they are reflected in both the social welfare function (SWF), which aggregates individual utilities (Jenkins and Van Kerm 2012), and the corresponding inequality indices.

In general, individual utility of income is assumed to increase with income at a decreasing rate. Conventional approaches to model the utility of income are based on the following parametric specification (e.g. Layard et al. 2008):

$$u = \begin{cases} \dfrac{y^{1-\rho}}{1-\rho}, & \text{if } \rho \neq 1, \\ \ln y, & \text{if } \rho = 1. \end{cases} \tag{1}$$

The parameter ρ represents the elasticity of marginal utility with respect to income. y denotes own income, which is typically adjusted for household size and composition using an equivalent scale.

[1] The OECD scale has an equivalence scale elasticity between 0.53 and 0.66 (Schwarze 2003).

The utility function in equation 1 is clearly restrictive, as it does not allow for reference-dependence, for instance. However, previous studies provide extensive evidence for comparison income effects (e.g., Luttmer 2005, Ferrer-i-Carbonell 2005, Clark and Senik 2010). Therefore, the utility of own income appears to depend on a comparison income that defines a reference point. This implies that not only the *level* of own income but also *deviations* of own income from the reference point may be relevant for utility. In order to capture reference-dependence, we specify an additional component for the utility function:

$$v(y,\tilde{y}) = \begin{cases} \gamma^{+}\left(\dfrac{y^{1-\rho}-1}{1-\rho} - \dfrac{\tilde{y}^{1-\rho}-1}{1-\rho}\right)^{\delta} & \text{if } y \geqslant \tilde{y} \\ \gamma^{-}\left(\dfrac{\tilde{y}^{1-\rho}-1}{1-\rho} - \dfrac{y^{1-\rho}-1}{1-\rho}\right)^{\delta} & \text{if } y < \tilde{y}, \end{cases} \tag{2}$$

where $\tilde{y}$ denotes the reference income. Following Ferrer-i-Carbonell (2005), we define the reference income, $\tilde{y}$, as the average income of a reference group that consists of individuals with similar educational attainment, of the same age group, and living in the same region.[2] The parameter δ, for which we assume $0 \leqslant \delta \leqslant 1$, allows for diminishing sensitivity of gains and losses, as postulated in prospect theory (Kahneman and Tversky 1979). Finally, the parameters γ^{+} and γ^{-} allow for differential effects of gains and losses, respectively.

3 Estimation strategy

We specify a regression model that takes account of behavioral and social factors that may determine the utility of income. In particular, we model reference-dependence and preferences about inequality. The regression model is based on equations 1 and 2 to estimate the utility of person *i*

[2] As an alternative, we assume that reference income is determined by the individual's own past income, as in Jäntti et al. (2014).

at time t as

$$s_{it} = \mathbf{x}'_{it}\boldsymbol{\beta} + \alpha h(y_{it}, \tilde{y}_{it}) + \eta_i + \varepsilon_{it}, \tag{3}$$

where

$$h(y_{it}, \tilde{y}_{it}) = \begin{cases} \frac{y_{it}^{1-\rho}-1}{1-\rho} + \gamma^{+} \left(\frac{y_{it}^{1-\rho}-1}{1-\rho} - \frac{\tilde{y}_{it}^{1-\rho}-1}{1-\rho} \right)^{\delta}, & \text{if } y_{it} \geqslant \tilde{y}_{it} \\ \frac{y_{it}^{1-\rho}-1}{1-\rho} + \gamma^{-} \left(\frac{\tilde{y}_{it}^{1-\rho}-1}{1-\rho} - \frac{y_{it}^{1-\rho}-1}{1-\rho} \right)^{\delta}, & \text{if } y_{it} < \tilde{y}_{it}. \end{cases}$$

The vector $\mathbf{x}$ is a vector of standard socio-economic control variables, α and $\boldsymbol{\beta}$ are additional parameters to be estimated. η represents individual-specific unobserved heterogeneity, and ε is an idiosyncratic error term. s is the experienced utility of income. We use data on subjective well-being to approximate utility (Frey and Stutzer 2002).

We carry out a three-dimensional grid search procedure to estimate the nonlinear-in-parameter structure of our model. The grid search approach is a flexible tool that, in our case, allows to control for unobserved individual fixed effects. The first dimension is the equivalence scale elasticity e, the second dimension is ρ, the rate of diminishing of the marginal utility of income, and the third dimension is δ, the diminishing of sensitivity for gains and losses. The first search was conducted in the intervals $e = [0(0.1)1]$, $\rho = [0(0.2)3]$, and $\delta = [0.1(0.05)0.9]$, a second refined search focussed on $e = [0.4(0.1)0.6]$, $\rho = [1.05(0.01)1.35]$, and $\delta = [0.45(0.005)0.8]$. The model selection is based on log likelihood values.

4 Data

We use data from the German Socio-Economic Panel (SOEP).[3] The SOEP is a longitudinal survey of households in Germany, run annually by the German Institute for Economic Research (DIW)

[3] We use SOEPv31.1 (DOI: 10.5684/soep.v31). For more information, see http://www.diw.de/soep.

(Haisken-DeNew and Frick 2005, Wagner et al. 2007). It covers micro-data on demographic, economic, social and political topics, including questions on subjective well-being.

The sample used in this study consists of individuals who are at least 16 years of age and live in private households. Our window of observation covers the period 2003 to 2014, i.e. the period after the introduction of the euro cash, because we want to avoid potential confounding effects on financial satisfaction due to the euro cash changeover (Wunder et al. 2008). We apply the following sample selection criteria: we exclude the top and bottom 1% of the distribution of equivalized monthly net household income to avoid influential outliers. Further, we do not use information from the first interview due to panel and learning effects with regard to questions about subjective well-being (Ehrhardt et al. 2000). In addition, Frick et al. (2006) argue that data quality improves for income questions after the first interview. We exclude all observations with missing values in key variables as well as individuals with less than 5 observations in the period of analysis. The analysis data is an unbalanced sample of 17,398 individuals and 158,693 person-year observations. Table 1 represents descriptive statistics of the key variables.

[Table 1 about here]

We use self-reported information about satisfaction with household income (i.e. financial satisfaction) to measure utility of income, which is the dependent variable in our analysis. In the SOEP, financial satisfaction is surveyed at the beginning of the questionnaire using the following questionnaire item:

> How satisfied are you today with the following areas of your life? Please answer by using the following scale: 0 means "totally unhappy", 10 means "totally happy".
>
> How satisfied are you with your household income?

The key explanatory variable is household income. It is reported by the household head. The corresponding questionnaire item is:

> If you take a look at the total income from all members of the household: how high is the monthly household income today? Please state the net monthly income, which means after deductions for taxes and social security. Please include regular income such as pensions, housing allowance, child allowance, grants for higher education support payments, etc.

We deflate household income with yearly consumer price index, with baseline year of 2011. It is adjusted to differences in household size and composition using equivalence scales of elasticities from 0 to 0.9. Following Ferrer-i-Carbonell (2005) and Vendrik and Woltjer (2007), we compute the reference income as the expected value for household income for given age, education, household size, region of residence in each year separately.

We control for a standard set of socio-economic background variables in the regression, including age squared, years of education, marital status, presence of children in the household, dummy variables to indicate if a person is employed, unemployed, lives in East Germany, lives in an owned home, health is controlled for with degree of disability and the number of nights in hospital in the previous year, finally year dummies.

5 Results

In this section, we present the results from our three-dimensional grid search procedure. The likelihood over the whole range is concave with a well defined maximum, as shown in Figures 1, 2, and 3. Table 2 shows estimates of the coefficients of the control variables. All regressions control for individual fixed effects.

[Table 2 about here]

The grid search yields the following maximum likelihood estimates of the nonlinear-in-parameter function. The maximum likelihood (ML) for the equivalence scale elasticity if $e = 0.5$,

which confirms previous findings, for example, in Biewen and Juhasz (ming). Our ML-estimate of the rate of diminishing of the marginal utility of income is $\rho = 1.2$. This result is close to the estimates obtained in Vendrik and Woltjer (2007) and Layard et al. (2008). Importantly, the estimate is above unity, implying that the marginal utility diminishes faster than income increases. The rate of diminishing of the marginal utility of income is equal to Atkinson's parameter of inequality aversion. Hence, the estimate may be used in the calculation of inequality indices from the Atkinson's family.

Loss aversion, conceptualized as the ratio of γ^- to γ^+, is not significantly different from 2. This confirms the seminal findings of Kahneman and Tversky (1979) and findings of Borah et al. (2016) using British and German survey data. Finally, the estimate of $\delta = 0.5$ indicates that the sensitivity for gains and losses diminishes with increasing distance from the reference point.

Next, we use our estimates to compute an income equivalent that explicitly takes into account the welfare effects of these behavioral factors. Following Jäntti et al. (2014), we calculate an alternative equivalent income, y^*, from the estimation results, which is defined "as the income level with which the individual would be equally well off, evaluated using a standard concave utility function, as he actually is, evaluated with a reference-dependent utility function" (p. 183). Plugging in the estimates, we compute the utility of income for each individual each year, i.e. $h(y)$. From these utility values we recompute an income equivalent, y^*, the income equivalent that would produce the same utility as the actual incomes, given the distance to the reference income. Formally,

$$h(y) = u(y^*). \tag{4}$$

This income equivalent represents a basis for an alternative calculation of welfare measures that take account of reference-dependence. These measure may provide supplementary information to the conventional equivalized incomes used in official reports of inequality.

Our results from the alternative equivalent income, y^*, show that utility of income appears to be generally smaller in the presence of reference-dependence. Table 3 illustrates. Effects of reference-dependence are particularly pronounces at the lower parts of the income distributions. Table 4 shows inequality indices and poverty measures that are calculated from the new equivalent income (i.e. taking account of reference-dependence).

[Table 3 about here]

[Table 4 about here]

6 Conclusion

A consensus on the measurement of economic welfare has not yet emerged. Different inequality indices that are based on different income concepts are used in the public debate on economic welfare. The purpose of this study was to scrutinize how the calculation of equivalent incomes may take account of behavioral and social factors that determine the utility of income. Extending the utility function of income enables us to provide a more realistic picture of (perceived) economic welfare than traditional indices that are not informed by insights from behavioral economics.

Using individual level data from the SOEP covering the years 2003 to 2014, we used self-reported measures of financial satisfaction to estimate an extended utility function of income. In general, our results do not indicate strong evidence for reference-dependence. With regards to income inequality, our new equivalent income does not dramatically change the conventional picture of economic welfare. However, poverty increases by 15 to nearly 30% when behavioral factors are taken into account. Thus, measures of economic inequality and poverty should take account of behavioral and social factors to provide a more complete picture of economic welfare.

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Tables and Figures

Table 1
Descriptive statistics

Variable	Mean	S.D.	S.D. within[1]	Min.	Max
Financial satisfaction	6.391	2.208	1.339	0	10
Household equivalence income[2]	1680.998	796.326	379.545	353.907	5904.554
Reference Income	1721.195	584.939	191.763	.934	6887.976
Household size	2.581	1.191	.490	1	14
Age	51.730	16.260	3.036	17	100
Years of education	12.262	2.685	.367	7	18
N night in hospital	1.573	7.791	6.698	0	350
Disabled	.138	.345	.161	0	1
Disabled up to 20%	.007	.082	.059	0	1
Disabled 21 to 40%	.032	.177	.111	0	1
Disabled 41 to 60%	.052	.222	.121	0	1
Disabled 61 to 80%	.023	.151	.085	0	1
Disabled 81 to 100%	.021	.145	.076	0	1
Living in owned home	.553	.497	.186	0	1
Female	.527	.499	0	0	1
Single	.176	.381	.184	0	1
Married	.746	.435	.183	0	1
Unemployed	.057	.232	.169	0	1
Employed	.577	.494	.250	0	1
Living in East Germ.	.262	.440	.061	0	1

Notes: [1] The within standard deviation describes the variation around the observation unit-specific mean, which is estimated as $\sqrt{(x_{it} - \bar{x}_i)^2}$ with $\bar{x}_i$ being the observation unit's mean. [2] Equivalent income is the monthly net household income, deflated with the square root of the household size to approximate the OECD equivalence scale.
Source: SOEP 2003 - 2014. 158693 person-year observations, 17217 individuals.

Table 2
Results from fixed effects regression of financial satisfaction

With equivalence scale elasticity $e = 0.5$;$\rho = 1.2$; $\delta = 0.5$

Fin. satisfaction	**Coef.**	**S.E.**	$P > \|t\|$	**[95% Conf.**	**Interval]**
U(y)	6.243	.141	0.000	5.965	6.522
Loss*V(y)	-.227	.087	0.009	-.399	-.055
V(y)	-.170	.058	0.003	-.284	-.056
Age^2	.000	.000	0.000	.000	.000
Years of edu.	.009	.010	0.332	-.009	.029
Employed	.13	.015	0.000	.102	.162
Unemployed	-.716	.022	0.000	-.759	-.673
Married	.208	.019	0.000	.169	.246
Children in hh	.09	.016	0.000	.059	.124
Disabled up to 20%	-.077	.058	0.186	-.191	.037
Disabled 21 to 40%	-.154	.031	0.000	-.217	-.092
Disabled 41 to 60%	-.147	.030	0.000	-.206	-.088
Disabled 61 to 80%	-.064	.042	0.136	-.148	.020
Disabled 81 to 100%	-.064	.047	0.174	-.157	.028
N night in hospital	-.003	.000	0.000	-.004	-.002
Living in East Germ.	-.353	.056	0.000	-.464	-.242
Living in owned home	-.016	.009	0.078	-.034	.001
Year fixed effects			yes		
Constant	-18.308	.590	0.000	-19.465	-17.150

N=158,693; n = 17,217; Source: SOEP v.31.1; Own calculations.

Table 3
Comparison of equivalized income y_{it} and income equivalent $y*_{it}$

Percentage overstating of utility through y_{it} compared to $y*_{it}$
For different positions in the income distribution

Year	Minimum	2. decile	2. quartile	Median	4. quartile	Maximum
2003	12.192	7.498	5.498	3.922	3.696	4.444
2004	12.323	7.031	7.234	4.568	3.497	2.683
2005	12.227	8.054	7.127	4.092	2.718	2.272
2006	11.445	8.868	6.332	5.272	4.205	2.678
2007	8.5681	8.538	7.147	5.264	2.565	4.526
2008	13.488	8.874	8.602	5.842	2.435	3.144
2009	10.108	8.341	6.969	5.022	3.434	2.865
2010	11.989	8.556	5.327	4.615	3.570	2.874
2011	12.570	5.476	5.314	3.761	2.590	4.034
2012	13.713	8.555	4.417	3.837	2.449	3.290
2013	11.989	5.443	5.195	5.389	2.894	4.230
2014	11.774	3.615	6.470	3.700	3.139	4.567

N=158,693; n = 17,217; Source: SOEP v.31.1; Own calculations.

Table 4
Comparison of Inequality and Poverty in the distributions of y and $y*$ in 2014

Measure (M)	y	$y*$	$\frac{M(y*)-M(y)}{M(y)}$
Measures of income inequality:			
Relative mean deviation	.173	.179	.038
Coefficient of variation	.444	.461	.037
Standard deviation of logs	.438	.458	.045
Gini coefficient	.241	.251	.038
Atkinson measure (e=1.2)	.105	.112	.062
Mehran measure	.339	.353	.038
Piesch measure	.192	.200	.038
Kakwani measure	.052	.056	.076
Theil entropy measure	.092	.100	.077
Theil mean log deviation measure	.094	.102	.085
Measures of income poverty:			
Headcount ratio %	6.990	8.096	.157
Poverty gap ratio %	1.008	1.298	.287
Thon index *100	1.973	2.532	.282
Sen index *100	1.412	1.798	.264
Takayama index *100	0.975	1.250	.281

N=158,693; n = 17,217; Source: SOEP v.31.1; Own calculations

Figure 1
Likelihood over equivalence scale elasticity

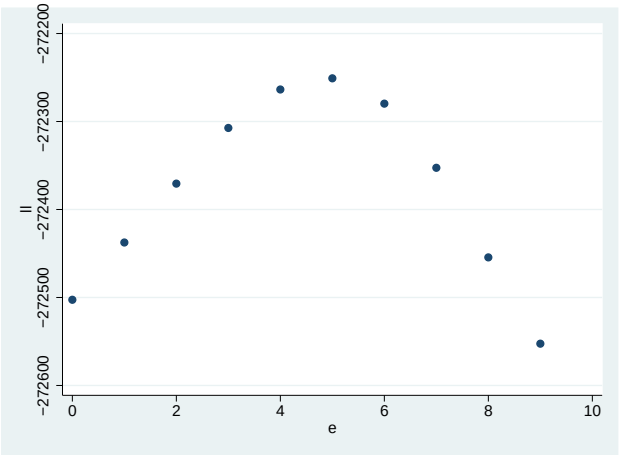

Figure 2
Likelihood over ρ

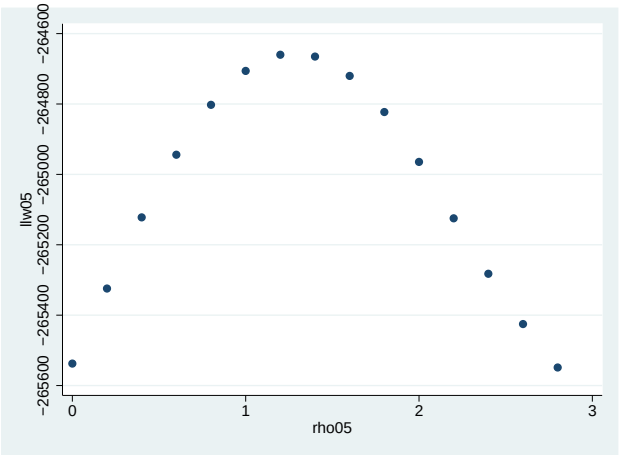

Figure 3
Likelihood over δ